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Coal Movement by Railroad in the Powder River Basin.

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INTRODUCTION

Coal is the leading supply for power generation in the United States. It is also the largest commodity transported on rail in the U.S., in terms of revenue and volume. Wyoming's *Powder River Basin (PRB)* (Figure 1, Figure 2) is the leading domestic supplier with a market share approaching 40%, due to its low sulfur and high BTU content, the low cost of *strip* mining, and the railroad network that moves it to power plants up to 1500 miles away. As Figure 3 illustrates, coal generates over 60% of the wattage consumed in the United States. Electrical power accounts for 95% of the total domestic coal consumption. Over 75% of domestic coal leave mines by rail, over 90% in the PRB. Coal transportation is also pivotal to the railroad industry as it accounts for over 40% of total freight tonnage and over 20% of total freight revenues [DOE, 2000].

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The Clean Air Act Amendments (CAAA90) have given economic incentive to power generating utilities to switch to low sulfur coal. As a result, PRB coal is now shipped over long distances (beyond 1500 miles in some cases, roughly 1000 miles on average). The resulting surge in demand for coal supplied by the ten mines in the southern PRB (127M tons in 1992, 283M tons in 2000; see Figure 4), and hence the additional pressure on its railroads, have exposed problems of efficiency and utilization on a once under-used, now congested network.

Additionally, the recent evolution of the industry has shortened supply chain contracts: newer ones are generally no longer than two years, with a trend towards *spot* (less than 12 months). The PRB mines have thus niched themselves as low cost, low sulfur coal providers, putting additional pressure on the *Orin Line*, the railroad trackage that services ten of the largest coal mines in the country (Figure 1).

A coal train is commonly referred to as a *unit* train, since no blocks of railcars is detached from nor attached to it between its origin and destination. In general, unit trains are not owned by a railroad but by the power companies that purchase the coal. They arrive at the PRB with a specific mine destination. As Figures 1 and 2 show, they connect with the Orin Line from the South (Bill) or from the North (Donkey Creek). A train waits at one those junctions for locomotives and crew. When a *mine slot* opens (a long enough future time interval at its mine destination), the dispatcher informs the train which enters the Orin Line. The BNSF (Burlington Northern Santa Fe Railway) manages operations on the line (dispatching and track maintenance), and both the BNSF and the UP (Union Pacific) supply locomotives, fuel, and crew to power the unit trains from their entry points to their mine destination and back out. Trains coming in from the North typically service Caballo, Belle Ayr, Cordero Rojo, and Coal Creek mines, and exit the Orin Line back north. Trains from the South service Antelope, North Antelope, Rochelle, North Rochelle, Black Thunder, and Jacobs Ranch. They also leave the Orin Line the way they entered.

Railroad capacity did not increase proportionally to the PRB mines' output. From 1993 to 2001, Orin Line coal output more than doubled (138% increase). Parallel segments were added on some portions of the line to allow improved synchronization ("meets and passes") and staging capacity was expanded at the entry points. Still, coal trains experience long delays. Those are attributed mostly to the randomness inherent in the local operation. In the absence of randomness, trains could be scheduled out of their entry point to and from their mine with significant precision and the congestion effect would be minimal, since it would only include the planned "meets and passes" of full and empty coal trains on the line.

However, randomness cannot be assumed out of these operations. For example, due to their weight alone, coal trains are very sensitive to weather patterns: wind gusts or rain often prevents them from making it up some grades, requiring additional locomotives which must be dispatched in real time. Those must negotiate their paths around other scheduled trains which must stop or even pull over on a staging track. This hinders the execution of smooth operations, as the planned efficiency and timeliness of sophisticated dispatching and scheduling systems cannot be achieved.

Another attribute that causes delays is the lack of coordination between the mines, the railroad, and the utilities. The railroad and mines obviously aims at maximizing their returns on assets. A mine would thus ultimately prefer to be loading coal trains at all times (unscheduled idle time causes expensive opportunity losses), and the railroad's goal is to maximize the rate at which it takes its trains out of the line's entry points to and from their pre-set mine destination. The mines thus have an incentive to signal time slots to the railroad that are tight and often unachievable, whereas the railroad often prefers controlling the number of trains on the line and thus delaying the dispatch of some of them to ensure that it is maximizing its overall throughput (and in so doing, potentially incurring idle time at a mine and delaying a train's delivery to its power plant).

The natural topology of the southern portion of the Orin Line (our focus) parallels that of a tandem network of queues. When couched in such a model, ideal traffic patterns (those that maximize the railroad's returns on assets) call for decreasing traffic as railroads service mines deeper in the line, assuming, along with simplifying structural assumptions, uniform commodity prices across all mines. Specifically, a train should visit a deeper mine only if the incremental transportation and congestion costs are lower than the current mine's. Traffic should thus be "V-shaped", with high intensity at the end mines (North Caballo and Antelope), and decreasing intensity as we move inside.

We observe that traffic and mining (Figure 5), and railroad infrastructure [BNSF, 2000] on the Orin Line from 1992 to 2001 deviate from such idealized solutions. Despite the deviations, the ideal traffic patterns nonetheless glean insight on how the railroads can price out the additional congestion induced by the larger mines (e.g. Black Thunder, Jacobs Ranch) located far inside the Orin Line, and the main drivers of its railroad traffic.

Figure 4 shows the total coal output for the mines between North Caballo and Antelope. We observe an on-going increase in total output over the nine year history (averaging 10% per year). The total, nine year, increase is 138% (303M tons in 2001 over 128M tons in 1992). The slight leveling off in 1997 and 2000 are due to demand reaching total capacity, mine closings, and mine mergers. Still, it is worthy to note that no significant pattern can be attributed to railroad expansion, as the congestion problem impacts train delays and efficiency, rather than mine output.

Figure 5 provides a distribution of the total output across the ten mines, aggregated in North, Middle, and South clusters. Some data are approximate, as some mines merged operations and some loaders were taken off-line. In addition, the two largest mines (Black Thunder and Jacobs Ranch) are connectedto the Orin Line on a shared spur, but data are reported separately for those.

Only from 1996 to 1999 do we observe the "V-shape" traffic pattern. Clearly, the railroad provides a *service* to the mines which contract with the utilities directly. Such agreements are a function of commodity prices and the mines' respective output capacity, and it is evident that the railroad cannot implement traffic patterns that are conducive to maximizing *their* return on assets.

Still, as the three upcoming models demonstrate, it is feasible for the railroad to price out to the utilities the additional congestion that the actual patterns entail. Currently the utilities and the railroad sign contracts that are relatively insensitive to congestion. However, as the trend towards spot purchases grows, it may make sense for the railroad to incorporate congestion costs into their contracts, through the imposition of a toll, for example. This would result in higher prices for visits to mines that are deeper on the line, such as Black Thunder or Jacobs Ranch.

Next, we couch the operations in an M/M/1 tandem queueing structure and derive three models "railroad by itself", "railroad and mines", and "joint design" each geared at analyzing the congestion effects and how the railroad can best manage it. We summarize our results and provide avenues for future research in the conclusion.

2. MODEL DESCRIPTION, NOTATION, AND ASSUMPTIONS

We couch the PRB operations in a queueing structure that parallels the natural topology of the Orin Line, for which we develop a generalized profitability function. We impose model assumptions on industry structure, pricing, capacity, and arrival/service times in order to extract intuitive results that are conducive to lower congestion effects.

We provide three derivations: "Railroad by Itself", "Railroad and Mines", and "Joint Design". We assume throughout that the utilities are price takers for railroad services. In the first model, the railroad maximizes profits by optimizing on the demand vector and observe that the solution is an *Economic Order Quantity (EOQ)* type rule [Silver, 1975]. Such an approach involves restrictive structural assumptions on the input as railroads do not have the flexibility to choose how to distribute the total demand across the Orin Line (utilities contract mines directly). We palliate this in the second derivation where we simplify the market by unifying the railroad and mines, and minimize the total system cost (railroad + mining). We verify sustainability by deriving the *user equilibrium* of this system, a solution concept drawn from traffic economics [Beckmann, 1956]. We mitigate the inefficiencies between the two approaches with a toll pricing scheme, the difference between marginal and average costs. We then provide a joint design derivation (akin to vertical integration) in which the railroad simultaneously solves for demand and capacity. We observe that its solution yields linear type rules as opposed to EOQ. Our notation is as follows (boldface for vectors):

V( ) represents the profit per unit time function from the network operations.

$\boldsymbol{\lambda}$ is the vector of arrival rates for each train type, where a type uniquely defines the mine destination of a train and its entry point (North or South). Let 1, 2,..., M be the types that enter from the North and visit, respectively, stations 1, 2,..., M; and M+1,...,2M be those that enter from the south and visit, respectively, stations 2M, 2M-1,...,M+1. A *station* is the trackage portion on the Orin Line that intersects with a spur that leads to a mine.

T(j) is the set of train types that travel on station j.

μ is the *processing* capacity of each station. We assume it linearly related to the mine's processing capacity (the number of trains it can fill per unit time).
M is the total number of mines.

$\boldsymbol{r} = (r_1, \ldots, r_M, r_{M+1}, \ldots, r_{2M})$ is revenue per train type per unit time.

c measures the cost of congestion per unit time.

k is the capital cost rate associated with mine j.

$\boldsymbol{\Lambda}$ is the resulting arrival rates per station.

$$\Lambda_j = \sum_{i \in T(j)} \lambda_i$$

Since we model the Orin Line as a network of tandem M/M/1 queues, we assume that the train inter-arrival times and mine processing times are exponential, that staging capacity is infinite, that there is one coal loader per mine, and that trains are serviced on a First Come First Serve basis.

Our performance measure (equation (1)) is a profit rate restricted to PRB activities. The revenue portion is a linear function of *throughput*: the number of trains served by the railroad per unit time multiplied by a destination specific railroad charge. The cost portion increases with *cycle time* and *inventory* and with capital invested in the network. Costs include locomotive depreciation, fuel, labor, and delay penalties. Coal car depreciation is excluded as the utilities own most of them. In 1998, the unit waiting cost, c, was estimated at $400 per hour per train [RDI, 1998].

$$V(\lambda,\mu) = r \cdot \lambda' - c \sum_{j=1,\dots,M} \left(\frac{\Lambda_j}{\mu_j - \Lambda_j} \right) - k \cdot \mu \qquad (1)$$

By Little's Law, we know that shorter cycles lower inventories [Hillier and Lieberman, 1990] and hence lower congestion costs. The bracketed portion of (1) is thus the average inventory (or the number of customers in a system) for an M/M/1 system with infinite queue capacity, First-Come-First-Serve queue discipline, an arrival rate of Λ_j and a service rate of μ_j.

Shorter cycles also reduce the likelihood of *blocking* and *starving* instances at the mines (lost opportunities for mines that could be loading but are inaccessible due to congestion). Maximizing profit is clearly more expedient to either maximizing throughput or minimizing cycle time since the congestion trade-offs are captured in tractable form. Also, evaluating profitability in rate form links to the traditional accounting measure of Returns on Assets (ROA): given the total assets involved in the operation, the earning rates they generate are the revenue (throughput multiplied by the revenue per train) minus the aforementioned costs. ROA is the ratio of returns generated by assets divided by their value.

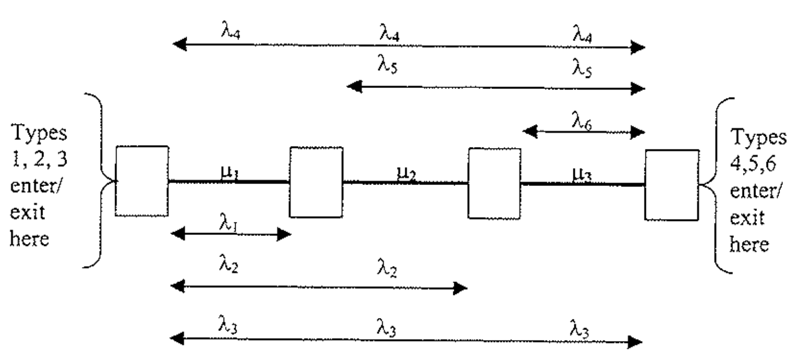
We model the Orin Line as a network of tandem queues. Locomotives and crew power the trains from the North (Donkey Creek) or South end (Bill) of the line to their preset mine. Once the train is loaded it departs from the mine and travels on the same (but in reverse order) set of tracks until it exits. Since each train has a fixed mine destination, that mine and the entry point of the train uniquely define the train's path in the network (the *visitation sequence* in a queueing network). We assign a unique number, the *train type*, to each possible path. To illustrate the movement suppose the network consists of four consecutive stations in tandem, numbered 1 to 4 from left to right, as per Figure 6. Train types 1, 2, and 3 enter from the left, visit stations in increasing order, and then follow the same path in reverse to eventually exit left. Types 4 to 6 enter from the right, visit stations in *decreasing* order, return and exit right. For example, type 4 trains enter right, visit track 4, then 3, then 2, then 2 again, then 3, then 4, then exit. Therefore, they travel twice on tracks 2, 3 and 4, and do not travel on track 1. For tractability, we assume out the spur portion from the Orin Line to the mine loaders, and focus on the traffic levels on the portions of the line *between* the mine spurs.

3. RESULTS: RAILROAD BY ITSELF

Maximizing (1) with respect to λ gives both optimal total traffic and its distribution across the M mines (or stations). We note that there are two possible train types per station, those that enter from the left and those that enter from the right. Trains that enter from the left are indexed 1, 2,..., M and those that enter from the right are indexed M+1, M+2,..., 2M. There are thus a potential of 2M train types in the network with station j being serviced by types j and 2M-j+1, j=1,...M. We assume that the revenue vector is increasing (i.e. $r_i > r_{i-1}$ for i = 1,...,M and $r_i > r_{i+1}$ for i = M+1,...,2M with r_0 and r_{2M+1} fixed at 0). This implies that the railroad charges incrementally, as a function of the distance the train must travel. In this model, the railroad is in effect pricing out the congestion effect on the initial segments caused by trains that visit mines deeper in the network.

Furthermore, we assume that the processing capacity at station *j*, μ_j, is fixed but sufficiently large. We also assume concave the revenue rate: the incremental revenues decrease as trains go deeper in the network,
i.e. $r_i - r_{i-1} > r_{i+1} - r_i$ for i = 1,...,M and $r_{i-1} - r_i > r_i - r_{i+1}$ for i = M+1,...,2M. The concavity assumption captures the fact that mines are somewhat clustered well inside Bill and Donkey Creek and that most congestion is encountered in the end segments. The cost is thus larger to reach the first mine from the entry point than it is to reach the second from the first. Furthermore, it reflects a "quantity discount" scheme which is prevalent in the provision of large scale services: the "per distance unit cost" to the railroad to pull a train, say, into mine 3 from mine 2 is assumed lower than into mine 2 from mine 1, since there are not as many trains on the 2-3 segment (namely, those destined to mine 2 from mine 1 turn around at mine 2 and never travel on the 2-3 segment).

To ease the exposition of the subsequent derivations, consider a three station representation of the line:



We solve

$$\max_{\lambda \geq 0} r \cdot \lambda' - c \sum_{j=1,\ldots,M} \left(\frac{\Lambda_j}{\mu_j - \Lambda_j} \right) \tag{2}$$

by deriving the Kuhn-Tucker conditions. The non-negativity constraints on λ implies, for the partial derivatives with respect to λ_1:

$$\lambda_1^* \left(r_1 - \frac{c\mu_1}{\left(\mu_1 - \left(\lambda_1^* + \lambda_2^* + \lambda_3^* + \lambda_4^*\right)\right)^2} \right) = 0 \tag{2a}$$

The partial derivative with respect to λ_2 yields

$$\lambda_2^* \left(r_2 - \left(\frac{c\mu_1}{\left(\mu_1 - \left(\lambda_1^* + \lambda_2^* + \lambda_3^* + \lambda_4^*\right)\right)^2} + \frac{c\mu_2}{\left(\mu_2 - \left(\lambda_2^* + \lambda_3^* + \lambda_4^* + \lambda_5^*\right)\right)^2} \right) \right) = 0 \tag{2b}$$

If λ_1^* and λ_2^* are positive, they are, respectively, the solution to

$$r_1 - r_0 = \frac{c\mu_1}{\left(\mu_1 - \left(\lambda_1^* + \lambda_2^* + \lambda_3^* + \lambda_4^*\right)\right)^2}$$ (with r_0 º 0)

and $$r_2 - r_1 = \frac{c\mu_2}{\left(\mu_2 - \left(\lambda_2^* + \lambda_3^* + \lambda_4^* + \lambda_5^*\right)\right)^2}.$$

The results are summarized in the table below.

Either	**Or**
$\lambda_1^*=0$	$\lambda_1^* + \lambda_2^* + \lambda_3^* + \lambda_4^* = \mu_1 - \sqrt{c\mu_1 / r_1}$
$\lambda_2^*=0$	$\lambda_2^* + \lambda_3^* + \lambda_4^* + \lambda_5^* = \mu_2 - \sqrt{c\mu_2 / (r_2 - r_1)}$
$\lambda_3^*=0$	$\lambda_3^* + \lambda_4^* + \lambda_5^* + \lambda_6^* = \mu_3 - \sqrt{c\mu_3 / (r_3 - r_2)}$
$\lambda_4^*=0$	$\lambda_1^* + \lambda_2^* + \lambda_3^* + \lambda_4^* = \mu_1 - \sqrt{c\mu_1 / (r_4 - r_5)}$
$\lambda_5^*=0$	$\lambda_2^* + \lambda_3^* + \lambda_4^* + \lambda_5^* = \mu_2 - \sqrt{c\mu_2 / (r_5 - r_6)}$
$\lambda_6^*=0$	$\lambda_3^* + \lambda_4^* + \lambda_5^* + \lambda_6^* = \mu_3 - \sqrt{c\mu_3 / r_6}$

Consider the first and fourth optimality conditions in the above table. It is unlikely that the railroad's pricing structure exhibits an exact result such as $r_1 = r_4 - r_5$ Therefore, we know that either $\lambda_1^*>0$; $\lambda_4^*=0$ or $\lambda_1^*=0$; $\lambda_4^*>0$. Moreover, if we assume **r** concave, then we should expect $r_1 > r_4 - r_5$ (since, on the Orin Line, the distance from the entry point to the first mine far exceeds the distance between any two adjacent mines). As we are maximizing (2), it should thus be the case that $\lambda_1^*>0$ and consequently that $\lambda_4^*=0$ (since $r_1 > r_4 - r_5$). Now suppose that at a generic station *j*, traffic comes from both sides. For this *j*, either $r_j - r_{j-1}$ is greater than $r_{2M-(M-j)} - r_{2M-(M-j)+1}$ or the reverse. The incremental revenue from servicing a customer on station *j* is thus larger if the customer comes from one side or the other. The implication of each pair of conditions for station *j* is that traffic at that station will only originate from the North or the South but never both.

If the above argument is valid, then we should also conclude that $\lambda_6^*>0$ and $\lambda_3^*=0$. It is thus the larger of $r_2 - r_1$ and $r_5 - r_6$ which will determine which of λ_2^* or λ_5^* is positive. Suppose that it is indeed λ_5^* (and thus $\lambda_2^* = 0$).

The fifth and sixth conditions above yields

$$\lambda_5^* + \lambda_6^* = \mu_3 - \sqrt{c\mu_3 / r_6} \quad \text{and} \quad \lambda_5^* = \mu_2 - \sqrt{c\mu_2 / (r_5 - r_6)}$$

from which we derive $\lambda_6^* = (\mu_3 - \mu_2) - \left(\sqrt{c\mu_3 / r_6} - \sqrt{c\mu_2 / (r_5 - r_6)}\right)$.
To further illustrate, suppose **r** = (4, 6, 7, 9, 8, 5). Observe that **r** is concave ($4>6-4>7-6$) and ($5>8-5>9-8$). Also, to simplify, we assume $\mu_i = \mu_j = \mu$ and that this processing capacity, the same at each station, is chosen to be large enough so that $\mu > \Lambda_j^*$ for all j. We also fix at one the unit congestion cost, i.e. $c=1$. The above table becomes

Either	Or
$\lambda_1^*=0$	$\lambda_1^* + \lambda_2^* + \lambda_3^* + \lambda_4^* \; = \mu - \sqrt{\mu/4}$
$\lambda_2^*=0$	$\lambda_2^* + \lambda_3^* + \lambda_4^* + \lambda_5^* \; = \mu - \sqrt{\mu/2}$
$\lambda_3^*=0$	$\lambda_3^* + \lambda_4^* + \lambda_5^* + \lambda_6^* \; = \mu - \sqrt{\mu}$
$\lambda_4^*=0$	$\lambda_1^* + \lambda_2^* + \lambda_3^* + \lambda_4^* \; = \mu - \sqrt{\mu}$
$\lambda_5^*=0$	$\lambda_2^* + \lambda_3^* + \lambda_4^* + \lambda_5^* \; = \mu - \sqrt{\mu/3}$
$\lambda_6^*=0$	$\lambda_3^* + \lambda_4^* + \lambda_5^* + \lambda_6^* \; = \mu - \sqrt{\mu/5}$

Since $r_1 > r_4 - r_5$ (i.e. $4>9-8$), it should thus be the case that $\lambda_1^*>0$ and consequently that $\lambda_4^*=0$. Analogously, we conclude that $\lambda_6^*>0$ and $\lambda_3^*=0$. It is thus the larger of $r_2 - r_1$ (=2) and $r_5 - r_6$ (=3) which will determine which of λ_2^* or λ_5^* is larger (λ_5^*).

We thus conclude that $\lambda^* = (\mu - \sqrt{\mu/4}, 0,0,0, \mu - \sqrt{\mu/3}, \sqrt{\mu/3} - \sqrt{\mu/5})$

with a maximized profit rate of

$$4\left(\mu - \sqrt{\mu/4}\right) + 8\left(\mu - \sqrt{\mu/3}\right) + 5\left(\sqrt{\mu/3} - \sqrt{\mu/5}\right) - \left(\sqrt{4\mu} + \sqrt{3\mu} + \sqrt{5\mu} - 3\right)$$

We now generalize our results. To simplify notation, let s_j represent the (post-optimal) incremental revenue to the railroad from pulling a train type into station j from either side. That is, s_j is the largest of $r_j - r_{j-1}$ and $r_{2M-(M-j)} - r_{2M-(M-j)+1}$.

Note that $s_j \geq 0$ for all stations and concavity of r implies that $s_j \geq s_i$ when i indexes a station deeper in the network than j. Since we solved with respect to λ, the total revenue to the railroad from pulling Λ^*_j trains into station j is either

$$\sum_{j_0=1}^{j} s_{j_0} \Lambda^*_{j_0} \text{ if the trains are coming from the left side,}$$

and $\sum_{j_0=j}^{2M} s_{j_0} \Lambda^*_{j_0}$ if trains are coming from the right side.

The optimal levels of demand per station can be expressed as

$$\Lambda^*_j = \mu_j - \sqrt{\frac{c\mu_j}{s_j}} \quad \text{or, in } \textit{safety capacity} \text{ form } \mu_j - \Lambda^*_j = \sqrt{\frac{c\mu_j}{s_j}}.$$

The right-hand sides are instances Economic Order Quantities which are known to be insensitive to small fluctuations in input [Silver, 1975].

Under our assumptions, we conclude (i) that the railroad prefers less traffic further into the network and (ii) that there should thus be at most one station with traffic originating from both ends of the network. The longer term effect of adjusting a rate schedule according to such results is that utilities gradually seek supply from mines closer to where their train enters the line, in effect establishing a geographical North-South partition which we now observe [DOE, 2000].

Also, the optimal demand levels can be expressed in safety capacity form (the difference between arrival rates of trains and the station's processing capacity). As those are in EOQ form, a robust connection to inventory management theory including the EOQ results' good approximation property and their insensitivity to perturbations in input. It thus follows that our allocation rule also exhibits such robustness. Moreover, comparative statics illustrate a positive relationship between incremental revenue and demand. This justifies the concavity assumption on **r** and makes the solution sustainable. Otherwise, the utilities would not be willing to supply more demand to stations deeper in the system.

4. RESULTS: RAILROADS AND MINES

In this section, we analyze solutions from the point of view of one entity that supplies both commodity and transportation to the utilities. In this model, the utilities purchase the coal *and* the railroad service that moves the coal to their plants. Hence, we include the commodity cost in addition to the transportation cost, and derive how a "central planner" would distribute a fixed amount of traffic across a simplified network in a way that minimizes total costs to the utilities (that is, total mine and railroad costs are minimized). In this model, we only look at trains entering the network from one end only and distribute a fixed amount of traffic intensity across two stations in tandem.

Let ***m*** represent the vector of mining costs (m_j is the cost to mine j to load one train). To ease illustration at no loss of generality, we exclude net profit components from this analysis, reduce the network to a network of two mines, fix $\lambda_1 + \lambda_2 = \Gamma$ and allocate Γ optimally between mines 1 and 2. The central planner must thus

$$\min_{\lambda_1,\lambda_2}(m_1 + s_1)\lambda_1 + (m_2 + s_1 + s_2)\lambda_2 + c\left(\frac{\lambda_1 + \lambda_2}{\mu_1 - (\lambda_1 + \lambda_2)} + \frac{\lambda_2}{\mu_2 - \lambda_2}\right) \quad (4)$$

where $\lambda_1 + \lambda_2 = \Gamma > 0$, $s_1 = r_1$, and $s_2 = r_2 - r_1$. The first-order condition implies

$$m_1 = m_2 + s_2 + c\left(\frac{\mu_2}{\left(\mu_2 - \lambda_2^{CP}\right)^2}\right) \quad (5)$$

which yields, in safety capacity form

$$\mu_2 - \lambda_2^{CP} = \sqrt{\frac{c\mu_2}{m_1 - (m_2 + s_2)}} \quad (6)$$

Observe that the structure of (6) requires $m_1 - (m_2 + s_2) > 0$ and, for λ_2^{CP} to be positive, $m_1 - (m_2 + s_2) > \frac{c}{\mu_2}$.

In traffic theory the user equilibrium (UE) solution will be such that, between a given origin-destination pair, the expected (average) cost on all paths that carry positive traffic be the same [Beckmann, 1956]. In the reduced network, this means that

$$m_1 + s_1 + \frac{c}{\mu_1 - (\lambda_1^{UE} + \lambda_2^{UE})} = m_2 + s_1 + s_2 + \frac{c}{\mu_1 - (\lambda_1^{UE} + \lambda_2^{UE})} + \frac{c}{\mu_2 - \lambda_2^{UE}}$$

which simplifies to

$$\lambda_2^{UE} = \mu_2 - \frac{c}{m_1 - (m_2 + s_2)} \tag{7}$$

Condition (7) differs from (5) as UE is based on average costs and the central planner (CP) on marginal costs. It is also important to note that the UE condition does not solve (4). This discrepancy implies an ordering between the CP and UE solutions when the savings are large enough:

$$m_1 - (m_2 + s_2) > \frac{c}{\mu_2} \Rightarrow 0 < \lambda_2^{CP} < \lambda_2^{UE} \tag{8}$$

A central planner would thus only send trains to the deeper mine if the mining costs there are lower than the additional railroad and congestion costs.
For example, suppose

$$(m_1, m_2; s_1, s_2; c) = (5, 3; 5, 1; 1),\ \mu_1 = \mu_2 = \mu \text{ and } \Gamma = \mu - 1.$$

We obtain

$$\lambda_2^{CP} = \mu - \sqrt{\frac{\mu}{5-(3+1)}} = \mu - \sqrt{\mu}, \text{ and } \lambda_1^{CP} = \sqrt{\mu} - 1$$

for a total cost of

$$(5+5)\left(\sqrt{\mu}-1\right)+(3+5+1)\left(\mu-\sqrt{\mu}\right)+\left(\frac{\mu-1}{\mu-(\mu-1)}+\frac{\mu-\sqrt{\mu}}{\sqrt{\mu}}\right)$$

which simplifies to $10\mu+2\sqrt{\mu}-12$.

The user equilibrium solution is $\lambda_2^{UE}=\mu-\frac{1}{5-(3+1)}=\mu-1$ and $\lambda_1^{UE}=0$ for a total cost of $11\mu-9$. Therefore, without a central planner approach, the railroad and mines entity prefers sending all traffic to the second mine $\left(\lambda_1^{UE}=0\right)$. To recover this inefficiency, the entity must thus impose a toll on all trains destined to the further mine.

The *user equilibrium* differs from the central planner approach. In fact, when the incremental savings of using the further mine exceed its minimum congestion cost, the central planner solution calls for less traffic at that mine than the user equilibrium solution. Basic micro-economics show that under fixed capacity marginal costs always dominate average costs beyond its minimum. Since this minimum occurs at zero traffic, inefficiencies are present. To mitigate those, we seek a pricing scheme (a user fee or a toll) that induces the users into a central planner solution. Such a toll is, exactly, the difference between marginal cost and average cost. In the numerical example above, we can recover the toll by charging a per unit fee to the trains that visit the second mine. The difference in total costs (between UE and CP) equals

$$11\mu-9-\left(10\mu+2\sqrt{\mu}-12\right)=\mu-2\sqrt{\mu}+3.$$

The toll per train should then be

$$\left(\mu-2\sqrt{\mu}+3\right)/\Gamma=\left(\mu-2\sqrt{\mu}+3\right)/(\mu-1)$$

We factored out of this analysis the profit margins of the supply side of the market. The above toll can be integrated in the profit margin as an additional side payment to the railroad for servicing trains at deeper mines. The toll for the simplified version of the network can easily be extended for the general version, again as the difference between the marginal and average cost of sending trains to a specific mine.

5. RESULTS: JOINT DESIGN

This analysis provides insight on how the optimal levels of capacity and demand interact with each other, as well as with the input to the problem. For this framework, we maximize (1) with respect to both λ and μ. The derivations parallel those of the "Railroad by Itself" section, but we distinguish from those univariate solutions by indexing the optimal variables here with two stars:

$$\lambda_i^{**} = 0 \text{ or } r_i - r_{i-1} = \frac{c\mu_i^{**}}{\left(\mu_i^{**} - \Lambda_i^{**}\right)^2} \text{ for i = 1,...M} \tag{9}$$

$$\lambda_i^{**} = 0 \text{ or } r_i - r_{i+1} = \frac{c\mu_{2M-(M+i)}^{**}}{\left(\mu_{2M-(M+i)}^{**} - \Lambda_{2M-(M+i)}^{**}\right)^2} \text{ for i = M+1,...,2M} \tag{10}$$

$$\mu_j^{**} = 0 \text{ or } k_j = \frac{c\Lambda_j^{**}}{\left(\mu_j^{**} - \Lambda_j^{**}\right)^2} \text{ for j = 1,...M} \tag{11}$$

We conclude first that at any station, it must be the case that the incremental revenue rate to visit that station must exceed its capital cost rate (the cost for increasing processing by one unit per unit time).

Furthermore, we can also rewrite the optimal levels of demand and processing capacity in safety capacity form and observe an inverse relationship between the railroad's profit margin at each station and the safety capacity at that station. In queueing, the *utilization* of a station, ρ, is the probability that this station is processing a customer, effectively measuring the station's value to the overall activities since ρ is also interpreted as the proportion of time the station is busy [Gross and Harris, 1974]. We factor (11) into (9) and (10) and conclude that, at optimality,

$$\rho_j = \frac{\Lambda_j^{**}}{\mu_j^{**}} = \frac{k_j}{s_j} \tag{12}$$

where s_j is the (post-optimal) incremental cost of pulling a train into station j. Since ρ must be less than one, (12) implies that at station j, it must be the case that the incremental revenue rate exceeds the cost rate, $s_j > k_j$.

Moreover, we observe $\mu_j^{**} = \frac{cs_j}{(s_j - k_j)^2}$ and $\Lambda_j^{**} = \frac{ck_j}{(s_j - k_j)^2}$,

or in safety capacity form: $$\mu_j^{**} - \Lambda_j^{**} = \frac{c}{s_j - k_j} \qquad (13)$$

When the margin is large, we expect optimal demand to be close to optimal capacity, otherwise there is waste of resources. When safety capacity is large, the unused capacity exposes the low profitability of that station. For sustainability, the concavity assumption on revenue rates must be combined to an ordering on the capacity costs that makes decreasing the profit rates as we go deeper in the network. Otherwise, the railroad and utilities would have conflicting incentives and this solution could not be realized.

CONCLUSION

Congestion, a natural outcome of the high traffic levels on the Orin Line and from the stochasticity of travel times on its segments, impedes railroad efficiencies by tying assets for unnecessarily long periods of time. As claimed in [State of Wyoming, 1992-2001], it is indeed the main stumbling block to more profitable operations on the Orin Line.

Railroad transportation is a large industry with broad sources of revenues. To properly gauge the performance of its coal movement activity, we developed a localized measure that reveals performance of a specific operation. It comprises of a linear revenue function (arrival rates), and a two-tier cost function (congestion and capital). The assumptions on exponential times allowed us to derive a closed (and convex) expression for the congestion portion.

Keeping capacity fixed, we derived traffic patterns conducive to lower congestion effects. Our approach for this was three-fold. Initially, we used the same revenue component and solved for demand patterns. The solution was in EOQ form, and a connection to Inventory Theory was established. Moreover, if the railroad's rates exhibit a scaled behavior, then optimal demand levels decrease as we move to stations further inside the network, inducing a "V-shaped" optimal traffic pattern with at most one station servicing traffic that originates from both ends of the network.

Since demand between utilities and coal mines is driven by mine prices, we cannot expect it to naturally induce low congestion costs to the railroad. In the second approach, we added the mining costs (linear as well) to the performance measure. To test sustainability of our analytical solution, we derived the user equilibrium condition from traffic theory. Since that solution is based on average costs, we remarked that when the incremental savings of using a mine deeper in the network exceed the minimum congestion cost of the current mine, a central planner solution (that minimizes total system costs) would call for lower traffic at the deeper mine than the user equilibrium solution. To mitigate this inefficiency, we computed a toll which is exactly the difference between the marginal and average costs.
In the third approach, we simultaneously solved for capacity and demand levels, with the mines excluded from the analysis. We derived closed form solutions for optimal safety capacity which are linearly related to each station's profitability margin.

Results in this vein can be extended to mirror more specifics of the operations at no cost to our analytical structure. For example, we assumed that utilities are price takers for railroad service and coal. In reality, utilities face downward sloping demand curves for such services. By parametrizing prices with demand curves, we could simultaneously derive optimal supply levels and prices for each coal mine, as well as optimal supply levels and rates for railroad service.

Extensions of interest would be to model cross ownership of mines, along with two railroad companies competing to provide service in the PRB. Refining the operation to an oligopolistic framework may yield results that exhibit a similar service structure as the ones obtained under independent mine ownership and a single railroad, and would also predict contractual or acquisitional behavior amongst the different parties involved.

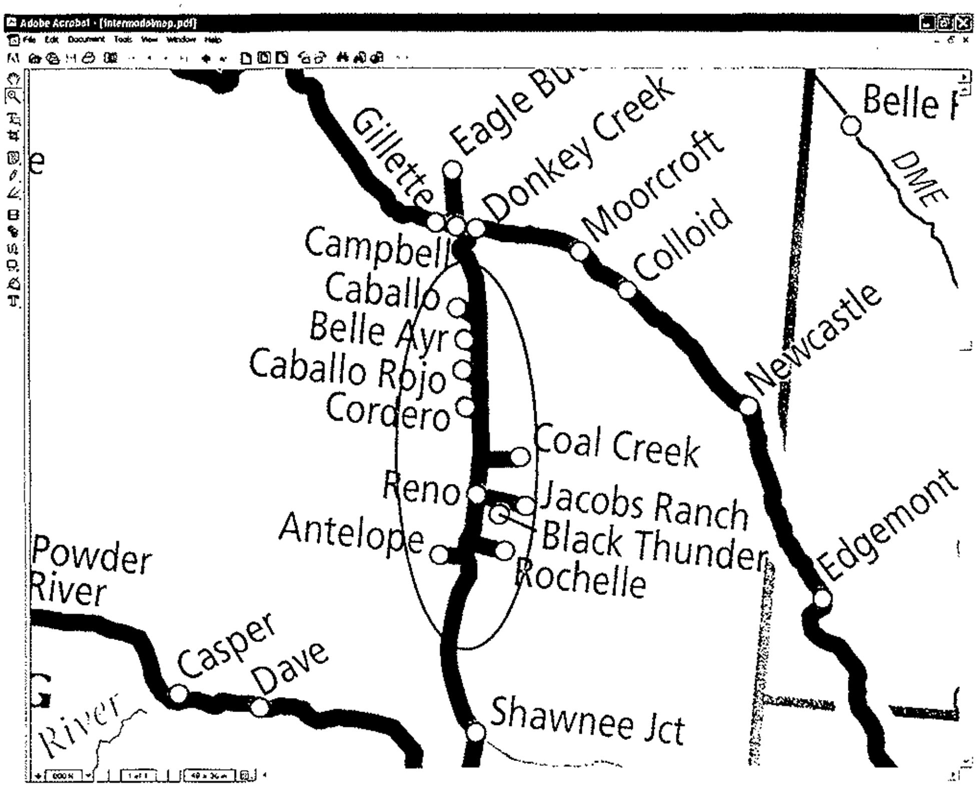


Figure 1: Railroad map of the Powder River Basin. (Source: RDI)

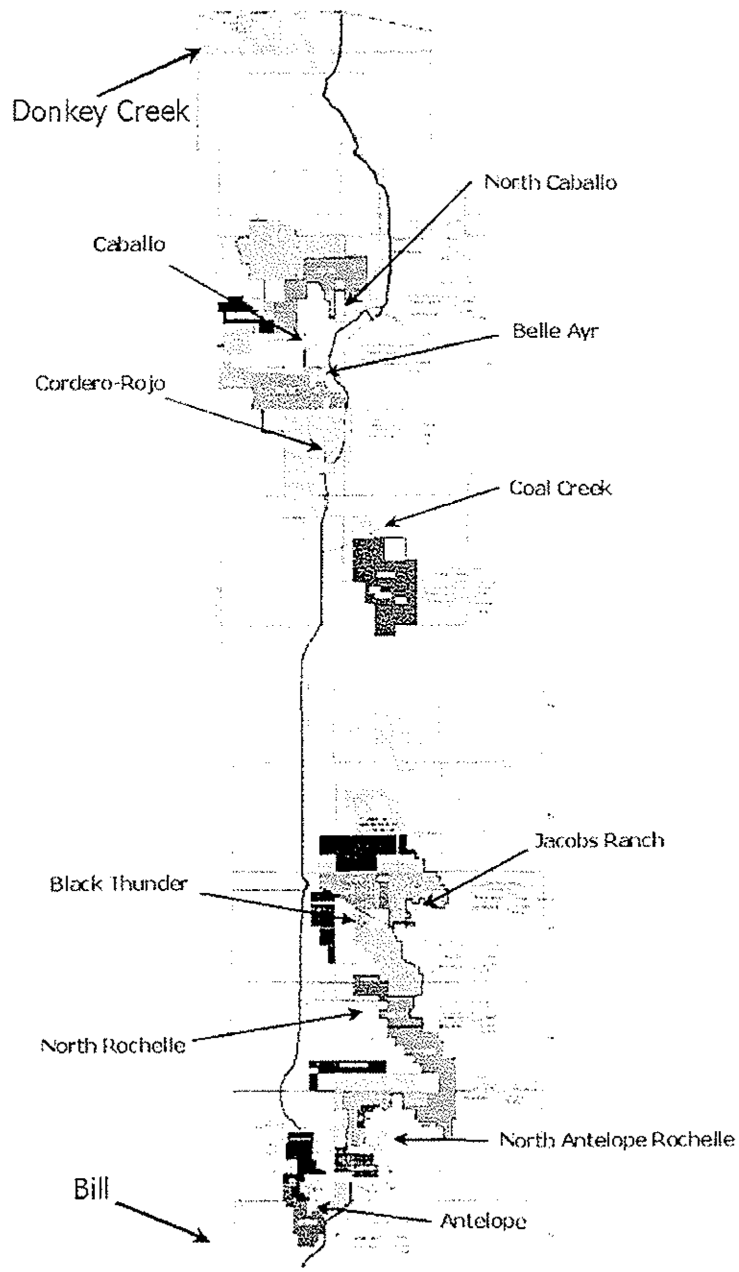
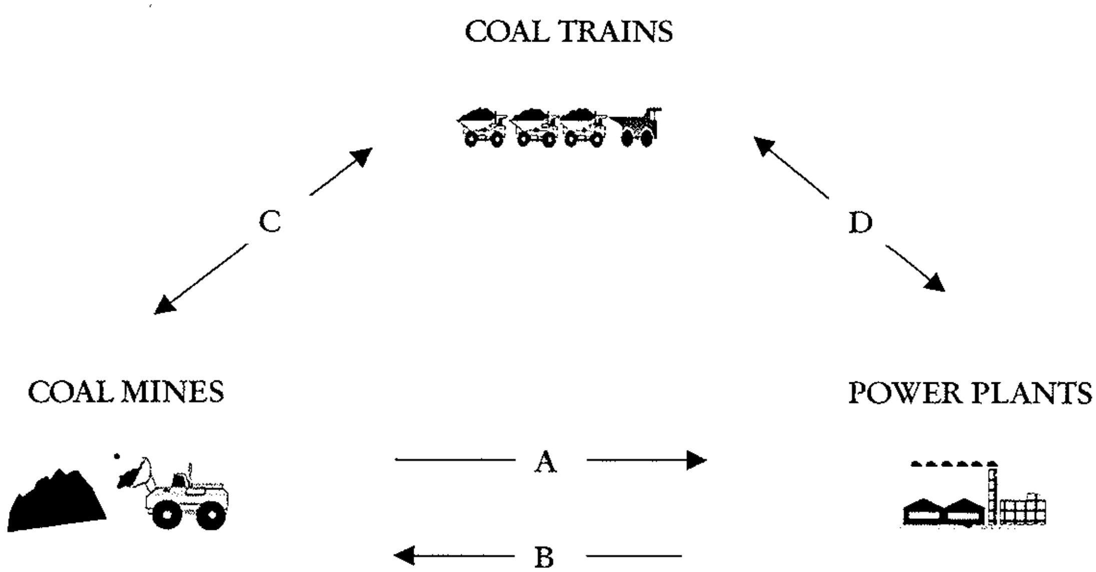


Figure 2: Mine map of the Southern Powder River Basin. (Source: Sate of Wyoming)



A: Coal generates over 60% of wattage in the U.S.
B: Electricity accounts for over 95% of coal consumption.
C: Over 75% of domestic coal leaves mines by rail (over 90% in the Powder River Basin).
D: Coal transportation accounts for over 40% of freight tonnage and over 20% of freight revenues.

Figure 3: Coal. Railroad, and Utilities

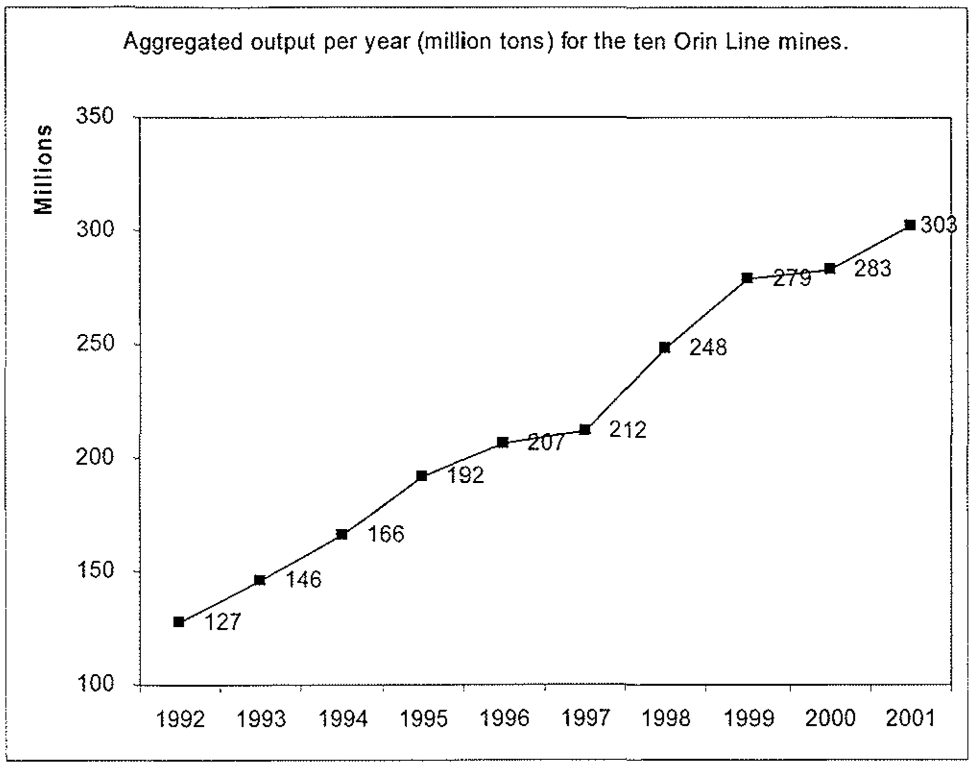


Figure 4: Total Output, 1992-2001, from North Caballo To Antelope. (Source: State of Wyoming)

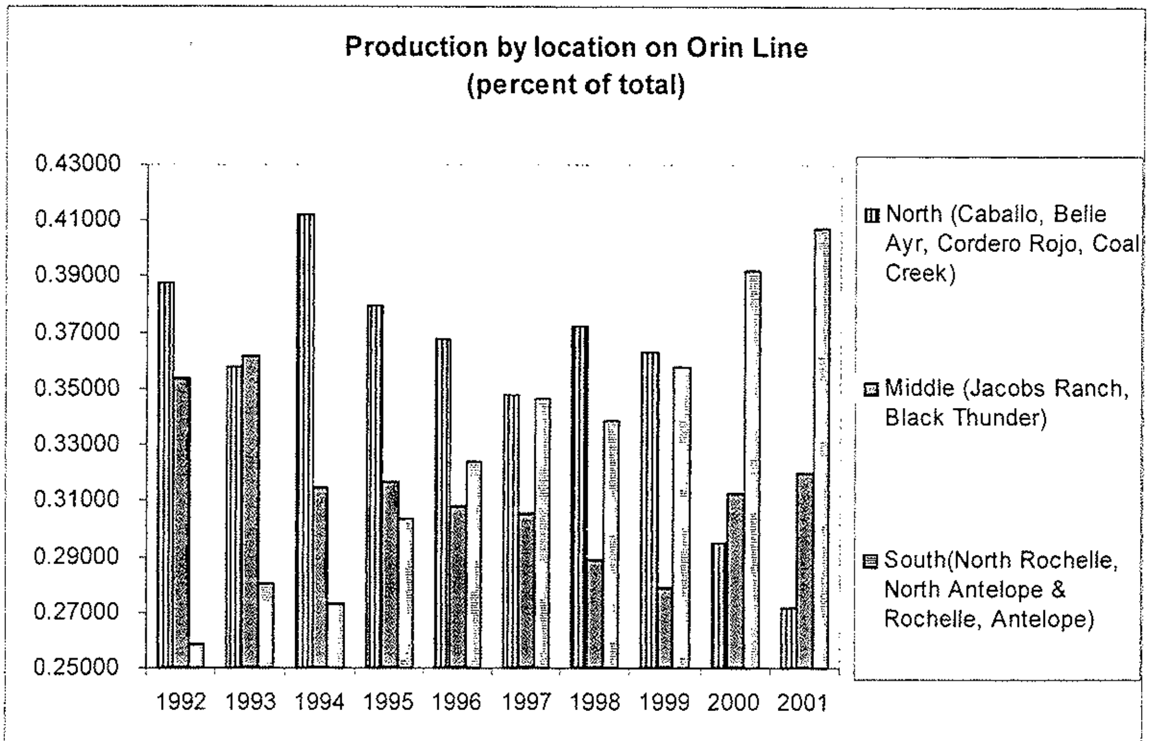


Figure 5: Total coal output per year, by mine clusters, 1992-2001. (Source: State of Wyoming)

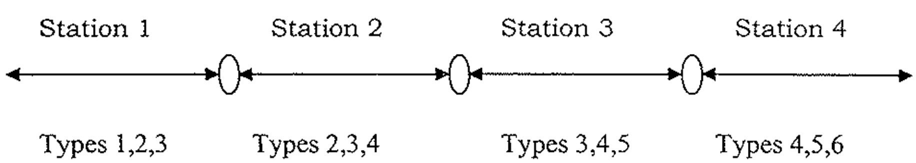


Figure 6: Train type and mine correspondence

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