A Review of Methods for Estimating Future Hydrocarbon Supply

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Techniques to estimate the size of the undiscovered hydrocarbon resource base and produce discovery rate forecasts are reviewed and assessed. Accurate assessments of the resource base are needed by exploration firms and the government in comparing the potential of competing areas of exploration and by policy makers in developing appropriate policies to address possible hydrocarbon shortages. The techniques reviewed include judgemental prediction, extrapolative methods, discovery process models, econometric models, and probabilistic models. A critical distinction exists between econometric and geologically-based approaches. The geological approaches aim at explicitly determining the size of the undiscovered resource base and its frequency-size distribution, producing discovery rate forecasts as a by-product of the adopted methodology. The econometric approaches aim explicitly at forecasting future discovery rates, producing estimates of the resource base as byproduct.

Cet article examine et évalue les techniques qui servent à estimer le volume des ressources non découvertes d'hydrocarbures et à faire des prévisions quant aux taux de découverte. Des évaluations précises du volume de la ressource sont utiles aux entreprises exploratrices et aux gouvernements pour comparer le potentiel de différents sites d'exploration, ainsi qu'aux décideurs pour préparer les mesures à prendre en cas d'éventuelles pénuries d'hydrocarbures. Les techniques examinées comprennent la prédiction au jugé, les méthodes d'extrapolation, les modèles économétriques et les modèles de découverte, les modèles économétriques et les modèles probabilistes. Une nette distinction est faite entre les méthodes économétriques et géologiques. Les méthodes géologiques, telles que les modèles probabilistes et ceux de découverte, visent expressément à déterminer la taille de la ressource non découverte et sa distribution fréquence-taille, d'où sont dérivées des prévisions de taux de découverte. Quant aux méthodes futurs de découverte, les estimations du volume de la ressource étant un résultant de la méthode adoptée.

Introduction

The estimation of the undiscovered resource base is possibly one of the most controversial and, because of the importance of petroleum products, one of the most important of all forecasting activities. The effort devoted to resource base estimation, however, has not substantially reduced the uncertainty surrounding the size of the remaining resource base or produced a consensus opinion as to which of the many estimation techniques is "best." Precise knowledge of the quantity of probable remaining resources is important to firms exploring for hydrocarbons and to government policy makers. The interest of hydrocarbon exploration firms stems from their need to compare alternative areas of interest for discovery potential and to forecast their associated discovery rates and costs. The interest of policy makers stems from their need to ensure adequate future supplies of oil and gas. Unfortunately, a complete description of an area's geology does not exist until the area

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has been completely explored. Then the information is of little use to either the firm or the policy maker. Accordingly, predictive models of a region's geologic potential must be developed.

A wide variety of predictive techniques has been developed. The available techniques vary from the basin and play level of analysis to continental aggregations and from detailed structural and process models to simple extrapolations and curve-fitting. They also range from geologic based attempts to estimate the "in-situ" resource base to the economic based estimates of supply. In-situ estimates attempt to assess the stock of the available resource, while supply estimates focus on the rate at which the resource base is depleted. Because of the consequences of overestimation which include, for the firm, the inefficient allocation of exploration resource efforts, and, for policy makers, complacency in encouraging the development of hydrocarbon substitutes, it is important to critically assess the techniques used in the literature to estimate the size of the undiscovered resource base. The purpose of this paper is to provide a review and assessment of the available techniques for interested energy analysts and others not trained in geology.

Judgemental Prediction Techniques

The simplest technique for predicting the probable size of the remaining resource base is geologic analogy. See, for example, Weeks (1950) and Klemme (1971). The technique assumes that the geologic characteristics of area A resemble those of area B. Area B, about which little may be known, is then assumed to demonstrate a similar pattern of hydrocarbon occurrence as area A, about which much is known. Scaling factors are used to correct for obvious differences between the two areas, but little else is done to expand the methodology. Its chief advantages are that it ties in well with accumulated geologic experience and is particularly useful for assessing the potential of frontier areas about which little is typically known.

More advanced geologic techniques include areal (Weeks, 1958 and 1975) and volumetric (Jones, 1975) methods. These techniques rely on field measurements of the hydrocarbon bearing sedimentary rock or on information obtained from drilling to predict the size of the potential resource base. The approaches break the estimation process into a number of smaller prediction problems. For example, the areal approach requires estimation of the area of exploratory interest, the percentage of the area likely to be productive and the yield per productive unit of area. Formally, the areal yield equation is as follows:

$$(1)\quad \textit{Areal Yield} = A \bullet P \bullet Y$$

A = *Area (hectares)*
P = *Productive percentage*
Y = *Yield per hectare*

The volumetric method (White and Gehman, 1979) expands on the areal technique by adding a factor estimating the average sedimentary thickness, or net pay, of the productive area to the equation used to compute expected potential. While improving on the areal technique, the method is difficult to apply with any accuracy prior to the completion of substantive drilling in the region of interest because of the requirement for the net pay estimate. To overcome this problem, data from similarly characterized and previously explored regions are used to infer the values required for the predictive relationship defined below:

$$(2)\quad \textit{Volumetric Yield} = A \bullet P \bullet N \bullet Y$$

A = *Area (hectares)*
P = *Productive percentage*
N = *Average net pay thickness (metres)*
Y = *Yield per hectare-metre*

Judgemental methods are simple to use and produce aggregate estimates of an area's potential. However, the need for more detailed data for prediction purposes limits their use to the geologic profession. In their simplest form they give no account of the expected discovery sequence or discovery size distribution. The tendency to discover large fields early in the exploration process (Root and Drew, 1979) demands some description of both factors if the predictions of the total quantity of oil and gas are to be properly used in policy analysis (Power and Jewkes, in press). Secondly, the use of geologic analogy in place of basin specific measurements reduces the technique to nothing more than a systematic method of prediction by analogy.

Attempts have been made to improve the methodology with the inclusion of Delphi techniques (White and Gehman, 1979) by averaging the opinions of several experts. Ivanhoe (1984) has argued that carefully controlled geological consensus estimates from a jury of experts "provide realistic evaluations of remaining resource potential." Questions remain, however, about the appropriate composition of the jury given that, "wherever explorationists know the local problems they are more pessimistic about finding new oil, and petroleum engineers' jobs tend to make them more cautious than exploration geologists." This suggests that volumetric forecasts remain plagued by the problems of anchoring, i.e. they tend to be too heavily influenced by current levels of industry optimism or pessimism (Power, 1990). Furthermore, as White and Gehman (1979) have argued, "the approach contains no documentation of the direct input and requires knowledge of how expert are the experts in order to assess the assessment."

Extrapolative Methods

Extrapolative methods have an important place in resource assessment because of their direct ties to the historical discovery record. One particularly important and popular extrapolative technique is rooted in the production history profile of fields. A great amount of attention has been paid to the work of M. King Hubbert (1965, 1966, 1967 and 1979). Hubbert relied heavily on the production history profile of fields to derive an extrapolative technique for forecasting probable cumulative discoveries. The approach assesses the time variation of three curves: the cumulative production curve, P, the proved reserves curve, R, and the cumulative proved discoveries curve, D. When plotted against time, curves P and D exhibit the familiar form of an S-shaped growth curve. The discoveries curve logically predates the production curve, tending to lead it by a constant amount, Δt, over the mid-range of both curves. At the beginning and end of the field's life cycle, proved reserves, R, equal zero. Thus, at $t = 0$ and $t = \infty$ the values P and D are equal and over the range $0 < t < \infty$ they will deviate from each other by the amount R. Accordingly, we may define the equation for the interrelationship between the curves by the simple expression:

$$(3) \quad D = P + R$$

The relationship among the rates of discovery, production, and the change in proved reserves is expressed by taking the time derivative of equation (3) as follows:

$$(4) \quad \frac{dD}{dt} = \frac{dP}{dt} + \frac{dR}{dt}$$

When the rate of change of proved reserves peaks, $dR/dt = 0$ and $dD/dt = dP/dt$. This allowed Hubbert to define a set of curves, depicted in Figure 1, whose end- and midpoint relationships were known.

Hubbert then observed the typical length of the lag between the cumulative discoveries and production curves, Δt, and used it, in combination with the cumulative production curve, to predict the probable date of peak production. By assuming that the peaks in both curves occur halfway through their respective cycles, he was able to produce a forecast of total cumulative discoveries and production.

The assumption is implemented by assuming that the S-shaped cumulative discovery and production curves have the functional form specified by the logistic curve equation (5) below.

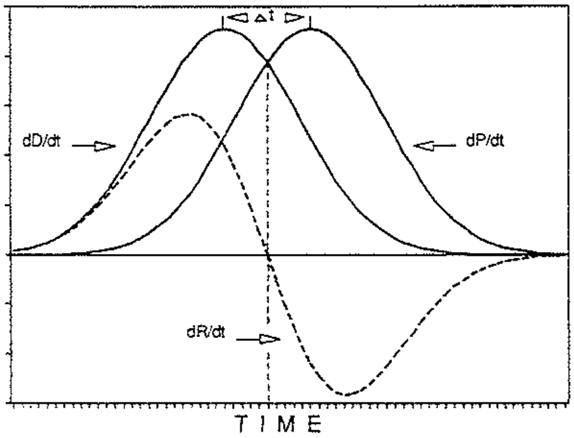

Figure 1: Hubbert's Curves for the Rates of Discovery, Production and Additions to Proved Reserves.

Depiction of the time-dependent interrelationship between the proved discoveries, production and the rate of increase in proved reserves as envisaged by Hubbert. Note the symmetry in the proved discoveries and production curves and the fact that the proved discoveries lead production by Δt over the mid-range of the two curves.

$$(5) \quad Q_t = \frac{Q\infty}{1 + \alpha e^{-b(t-t_0)}}$$

where

$Q\infty$ = *ultimate recoverable discoveries*
Q_t = *cumulative proved discoveries (or production) at time t*
α, b = *parameters to be estimated (both > 0)*
t_0 = *an arbitrary reference time (different for P and D)*

The curve is symmetrical about its own inflection point which is defined by $t = t_0 + (\ln\alpha)/b$ and $Q_t = (Q\infty/2)$. The parameter α determines where the curve will be located on the time axis and the parameter *b* determines the slope of the exponentially rising portion of the curve.

In Hubbert's work, Q∞ was estimated using a trial and error graphical approach in which the available data were plotted on semi-log paper and the Q∞ giving the best fit was selected by eye. It is now more common to linearize equation (5) to get:

$$(6) \quad \ln[(Q\infty/Q_t) - 1] = \ln(\alpha) - b(t-t_0)$$

and to estimate equation (6) by ordinary least squares for various values of Q∞. The Q∞ producing the highest R^2 is then selected as the best estimate of ultimate recoverable discoveries.

Uri (1980) points out that "one of the disadvantages of the logistic specification is that it implies symmetry with respect to time." The objection to symmetry lies in the fact that discovery rate curves are less likely to fall as fast as they grow. As it becomes clear that discovery rates are falling, operators are increasingly likely to invest in methods to improve the efficiency of exploration, especially if falling rates are perceived as a signal of impending scarcity and give rise to price increases. Such investments tend to make the final additions to cumulative discoveries stretch out over a longer period of time than the time required to make the initial discoveries.

A second fact arguing against the likelihood of symmetry with respect to time is the observed tendency of discovered pools to decline in size as the exploration process matures. For examples of this phenomenon, see Ryan (1973), Drew (1975) and Root and Drew (1979). The decreasing size of the discoveries implies that the distribution of those discoveries with respect to time is likely to be positively skewed, as Uri (1980) suggests.

Such objections have led many to suggest, including Moore (1971) and Uri (1980), that a more appropriate form of the S-curve for the purposes of prediction would be the Gompertz Curve. The curve is defined by an equation of the form:

$$(7) \quad Q_t = Q\infty \, exp[-b \, exp[-k(t-t_0)]]$$

where

$Q\infty$= *ultimate proved discoveries*
Q_t = *cumulative proved discoveries at time t*
b,k = *parameters to be estimated*
t_0 = *an arbitrary reference time*

Unlike the logistic curve, the Gompertz curve is not symmetric with respect to time. Its inflection point is defined by $t = t_0 + (\ln b)/k$ and $Q_\infty = Q_\infty/e$. Thus, the estimated Q_∞ will be roughly 2.72 times the value of the cumulative discovery curve at its point of inflection. The shape of the curve more closely replicates the phenomenon of discoveries through time. However, there is no *a priori* reason why discoveries should mimic its functional form. It remains only an empirical description of the discovery-time relationship.

One of the most serious criticisms of the type of curve-fitting prediction analysis embodied in the use of the logistic and Gompertz curves is that there is no clear reason why any of the available functional forms should be selected. In discussing the techniques, Schuenemeyer (1981) points out that, " ... discovery-production extrapolation models are easy to apply and seem to work well at high levels of aggregation, especially in relatively mature areas. The predictions are of course a function of the mathematical form of the model ...." Shanz (1978) concurs when he states that " ... despite the vigour with which Hubbert examines past behaviour and projects the various patterns of US oil discovery and production into the future, this does not necessarily indicate that the logistic curve is a more reliable predictor of the future than any other curve that might have been chosen. The use of mathematical formulas to project trends forward provides an aura of precision and objectivity. However, the process of fitting and projecting is a more subjective process than it might appear. The choice of the type of curve to be used pre-ordains in a general way what the future will look like." Other methods, discussed below, avoid making projections as simple functions of time through the explicit inclusion of geologic or economic factors in the modelling process. Finally, as a forecast technique, the approach does not perform well in comparison to other methods in partially explored basins (Power and Fuller, 1992).

Discovery Process Models

Discovery process models differ from extrapolative methods in that they explicitly include factors that govern the process of discovering oil and gas as fundamental model assumptions. Critical to this class of models is that large fields tend to be discovered early in the exploration process. See, for example, Arps and Roberts (1958), Drew (1975) and Root and Drew (1979). This phenomenon is incorporated by assuming that the chance of discovering an additional field of a given size is proportional to the number of undiscovered fields of that size remaining and the areal extent of the field. The main attraction of these models rests on the insights which they provide into the oil and gas discovery process and the evolution of discovery sizes through time. The models explicitly reflect the phenomenon of physical exhaustion with decreasing average discovery sizes as exploration progresses.

This approach was first proposed by Arps and Roberts (1958) and relied upon the observation that in the discovery history of the Denver-Julesburg basin a small proportion of the fields had contributed more than 50% of the discovered reserves. The majority of the discovered fields contributed only a small proportion of the discovered reserves. A plot of the frequency-size distribution of discoveries confirmed a log-normal distribution of field sizes. After breaking the distribution into distinctive field class sizes, Arps and Roberts proceeded to define the following general relationship describing the cumulative number of oil and gas fields of a given size that are discovered after the completion of w exploratory wells:

(8)

$$F_A(w) = F_A(\infty)\ [\ 1-e^{(-CAw/B)}]$$

where

$F_A(w)$ = *cumulative number of fields discovered in size class A by w wells*
$F_A(\infty)$ = *the ultimate number of fields in the area being explored*
B = *area of exploration interest*
w = *cumulative no. of exploration wells*
A = *average areal extent of the fields in a given field size class*
C = *the efficiency with which fields are discovered*

The expression incorporates the assumption that the additional number of fields discovered in a particular size class by incremental drilling activity "... must be proportional to the number of undiscovered fields " of the size class being considered and "... also proportional to the areal size of such fields." The model predicts the cumulative number of discoveries expected to be made within a given field size-class as a result of drilling w exploration wells.

The technique depends on obtaining estimated values for the parameters B and C. Values for B may be obtained by estimating the areal extent of the study area. Values for C are not as easily obtained. In their study of the Denver-Julesburg Basin, Arps and Roberts used an exogenous $C = 2.0$ estimate. Then, given C, B, and $F_A(w)$ for w wells, equation (8) may be solved backwards to obtain estimates of the ultimate number of fields expected in each field size-class as follows:

$$(9)\quad F_A(\infty) = \frac{F_A(w)}{[\ 1 - e^{-CAw/B}\ ]}$$

Now, however, it is common to determine C from available historical discovery data where values of $F_A(\infty)$ and C are chosen iteratively so as to minimize the difference between the observed and predicted values of $F_A(w)$ over a chosen historical interval (Drew, Schuenemeyer and Root, 1980; Drew, 1990). When the parameters C, B, and $F_A(\infty)$ have been estimated, the number of expected discoveries in the next increment of drilling activity, Δw, is determined from the solution to equation (8). The model produces a specific description of the ultimate field-size distribution and depicts how discoveries in each size-class will progress with time. In doing so it explicitly captures the phenomenon of exhaustion and declining average discovery sizes.

The model has been widely applied by the United States Geological Survey in making forecasts for the Denver Basin (Drew, Schuenemeyer and Root, 1980), the Permian Basin (Drew, Root and Bawiec, 1979), and the Western Gulf of Mexico (Drew, Schuenemeyer, and Bawiec, 1982).

Attanasi, Drew and Root (1981) have argued that, "... for petroleum exploration where the limiting factor is the finite number of large fields, forecasts should be based upon studies of the field size distributions. Prices and costs control economic behaviour but the amount of oil discovered is determined by the size distribution of the undiscovered fields. " There is no doubt that these models provide a powerful means of predicting future discovery rates and the mix of small and large discoveries. In that sense they give a clear view of the exploration process. However, model forecasts are conditional on the predicted future levels of exploration drilling. Great emphasis is laid on the model's ability to replicate the discovery process. Little, however, is said of how accurate forecasts of the parameter w are made, or can be made. Unless one is willing to regard future exploration drilling activity as pre-ordained, one cannot escape the issue of how uncertainties with respect to w will ultimately impact model predictions.

Furthermore, inaccuracies in the estimation of B and C parameters will also hold implications for the accuracy of model predictions. In particular, there is no reason to believe that the efficiency parameter C will remain constant throughout the drilling history of a basin. Tuning the selection of C to a particular historical data set runs the risk of allowing model predictions to be overly influenced by the lack or excess of recent drilling successes. As Ryan (1973) has noted, geologic learning increases with cumulative drilling effort and influences the recorded success ratios. Power (1992) estimated a series of drilling efficiencies for the Scotian Shelf and found them to be variable over time though, tending toward 2.0 as drilling progressed. The results suggest that at best discovery process models are limited in application to mature basins.

Econometric Models

Another class of resource base prediction methods are the econometric models. They approach the problem of resource base estimation by hypothesizing an economic theory of resource exploration and production and then empirically testing it against historical data. The econometric methodology traces its development to the pioneering work of Fisher (1964).

Fisher employed a simple three equation model to explain the number of exploration wells drilled per year, the proportion of wells that were successful (the success ratio), and the average discovery size. Each of the equations was of the form:

(10) $Y_{it} = f\ (\ P_t\ ,\ X_{it}\ ) \qquad i = 1, 2, \ldots , N$

where

P_t = *deflated wellhead resource price*
X_{it} = *structural variables*

and the X_{it}s were structural variables, including exploratory effort, depth, the past average discovery size, the success ratio, and various lagged dependent variables. The total discoveries made in any one period were calculated as the product of the three central equations. Thus the basic accounting identity in the model was:

(11) $D = W \bullet S \bullet Qd$

where

D = *discovered volumes*
W = *number of wells drilled*
S = *percentage of wells successful*
Qd = *average find per successful well*

Fisher distinguished between the extensive and intensive margins. At the extensive margin few large discoveries are made, while at the intensive margin many small discoveries are made. The observation led Fisher to specify both the success ratio and the average discovery size equations as functions of economic variables. Thus, in the short run, firms react to rises in price by shifting exploration from the extensive toward the intensive margin. The shift will tend to increase the number of discoveries that are made and to reduce the average discovery size.

MacAvoy and Pindyck (1973, 1975) followed Fisher by modelling new supplies using a similarly structured model. The success ratio and the average discovery size were in this case extrapolated from reference sizes and success ratios. Furthermore, the measures were sensitized to changes in price by basing the number of wells drilled on expected returns and the variability in returns, where returns were modelled as a function of the average discovery size and success ratio. A similar approach was also followed by Rice and Smith (1977).

Epple (1975) chose to model the oil and gas discovery process as a production process requiring wells and oil-bearing land as inputs. Exhaustion of the oil-bearing land resource was explicitly represented by the productivity of the land and the input cost of the land. Epple assumed that the required input of available oil-bearing land could be modelled as:

(12) $L = R^{B} e^{rw}$

R = *unit rent of the land*
w = *past exploration effort in number of wells*
$r, B > 0$ = *estimated parameters*

Given L, the optimizing behaviour of the firm is derived assuming the objective is the maximization of the net present value of exploration effort.

Uhler (1986) uses geologic information to sub-divide the Alberta basin into 10 homogeneous groupings of major stratigraphic intervals. The ratio of the rate of change in reserve additions to the rate of change in cumulative exploration effort is functionally related to cumulative exploration effort in an economic production function of the exploration process. Standard microeconomic techniques are then used to maximize the profit resulting from reserves additions and inferences about the price responsiveness of the reserve base made. While the approach takes the important step of recognizing the heterogenous geologic nature of areas as large as the Alberta basin, it nonetheless fails to incorporate into the modelling framework explicit considerations of the physical phenomena that control hydrocarbon occurrence and accumulation. As with all economic approaches, Uhler's objective of determining "how ultimate reserves respond to changes in the price of reserves" fails to recognize that at the extreme of exhaustion there is no response. Furthermore, the use of average drilling costs in completing the profit maximizing procedure renders the approach sensitive to the technological assumptions used to complete the analysis.

Economists have largely approached the resource base estimation problem simply as a market supply problem akin to (1) a firm's decision to invest in physical plant and equipment, as in the Fisher, MacAvoy and Pindyck approach, or, (2) as a basic production process where the number of prospects, exploration effort, and other relevant variables represent inputs into the hydrocarbon production process, as in the case of Epple. It is important to note that neither approach makes explicit reference to the physical factors controlling the occurrence of hydrocarbons. As Power and Jewkes (1991) have pointed out, econometric models do not take "specific account of the geologic parameters which inevitably control the resource exploration and development process." Indeed, economists tend to view the resource estimation problem as one of predicting the marginal increase in proved reserves resulting from an increment in either price or drilling effort, rather than as an exercise in determining the total size of the resource base.

Econometric modelling approaches have been attractive largely because they are consistent with and easily incorporated into the modelling frameworks available for completing policy analysis. The ability to construct a model specifically for policy analysis places energy policy issues squarely in the arena of public debate and helps to connect energy policy issues to the wider arena of macroeconomic interest. As Power and Jewkes (1991) have argued, traditional econometric models no longer provide an adequate means of assessing the attainment of stated resource development objectives because the objectives are now beginning to include quantity considerations. The lack of geologic detail in econometric models limits the understanding of the possible effects of any stated policy on resource exploration and development and demands the development of new policy tools.

Furthermore, there are a number of other problems with the econometric approach to resource base estimation. The industry supply response is based on the observed responses of individual firms under competitive conditions. There is no clear indication that such responses can be aggregated to accurately represent industry behaviour. As Bohi and Toman (1983) have pointed out "... the data base is plagued by errors, inconsistencies and confusion." Possibly worst of all, "... the estimation methods cannot separate dynamic interrelationships and identify separate influences." They tend to confuse notions of additions to the resource base with the commercial production from the discovered resource base. The former is clearly a function of exploration activity and the aim of resource estimation exercises. The latter is a consequence of changes in the policy or pricing environment. While such changes make more of the resource available for consumption they do not, and cannot, affect the process of discovery unless they directly influence the effort put into exploration.

Econometric approaches in no way account for the physical resource constraints so important to geologists and other physical scientists. As Drew (1990) has stated, economists hold the view that "there is nothing special about oil and gas. It is just a factor input to a production function." Geologists have, however, contended that it is the physical limits, not economics, that are the most important determinants of the resource base. Furthermore, they argue that these physical limit concepts should be explicitly incorporated into any resource base estimation technique.

A distinction then must be made between the short and the long runs. In the short run, price is more likely to determine supply, either because it accelerates production from existing discoveries or because it encourages the development of known marginally economic discoveries. In the long run, physical constraints come to dominate. Production will cease, irrespective of price, when exhaustion occurs. Furthermore, the frequency-size distribution of the discoveries constituting the resource base holds implications for the point at which, given price, new supplies are perceived as economic. A distribution dominated by many large discoveries implies cheaper supply than a distribution dominated by many small discoveries.

Geologists further criticize the econometric approach on the grounds that no rationale is given for the choice of many of the explanatory variables. What influence does the previous year's success ratio have on the average discovery size, or the average discovery size in year t-1 have on the average discovery size in year t? The models proposed to date proceed on the basis of observed correlation between the dependent and independent variables rather than on the basis of testable hypotheses. As Attanasi, Drew and Root (1981) have pointed out, this has resulted in the econometric models failing to reflect the finiteness of the resource base. Discovery sizes are not eventually forced to zero as they must be when exhaustion occurs. Despite these criticisms, the econometric approach remains popular and is likely to do so as long as it can provide at least a partial guide to the response of the resource base to fluctuations in the costs and price parameters of the hydrocarbon exploration, development and production process.

Probabilistic Methods

Probabilistic models are of three types: probabilistic discovery process models, statistical creaming models and simulation models. The probabilistic discovery process models include those developed by Barouch and Kaufman (1976), Smith (1980), O'Carroll and Smith (1980), Smith and Ward (1981), Lee and Wang (1983a, 1983b), Power and Fuller (1991) and Power and Jewkes (in press). The statistical creaming models include those developed by Meisner and Demirmen (1981) and Forman and Hinde (1985). Finally, simulation models include those developed by Roadifer (1975), White (1981), Procter *et al.* (1983) and Baker *et al.* (1984).

Probabilistic discovery process models explicitly include, as fundamental assumptions, factors that govern the discovery process for oil and gas. In that sense they are related to the deterministic class of discovery process models discussed above. The critical difference between the deterministic models discussed above and the probabilistic models is that the postulates made about the discovery process are probabilistic. Crucial to the models are that 1) the discovery of fields can be modelled as sampling without replacement from the underlying population of fields, and 2) the discovery of a field from the existing population is random with the probability of discovery being proportional to field size.

The resource base is modelled as consisting of J possible field sizes denoted as (S_1, S_2, ... S_J). The fields are assumed to occur with frequencies (n_1, n_2, ... n_J). The second postulate implies that the probability of discovering a field of any particular size will depend upon the frequency with which it occurs and the proportional importance of the field's frequency-size measure in the sum of all frequency-size measures for the population of fields being considered. More precisely, the probability that the first discovery D_1 will be of size S_j is given as:

$$(13) \quad P(D_1 = S_j) = \frac{n_j S_j}{\sum_{k=1}^{J} n_k S_k}$$

$$\text{for all } j = 1, 2, \ldots J$$

The number of fields of size S_j that remain to be discovered prior to the i^{th} discovery will depend directly on the discoveries which have occurred before the i^{th} discovery is made. The cumulative number of discoveries of the j^{th} size made prior to the i^{th} discovery is represented by the symbol m_{ij}. The probability that the i^{th} discovery will be of size S_j, conditional on the preceding set of discoveries, then becomes:

$$(14) \quad P(D_i = S_j \mid D_1, \ldots D_{i-1}) = \frac{(n_j - m_{ij}) S_j}{\sum_{k=1}^{J} (n_k - m_{ik}) S_k}$$

$$\text{for all } j = 1, 2, \ldots J$$

Given knowledge of the original deposition of the fields, it is possible to compute the probability of any particular discovery sequence occurring as the product of successive conditional probabilities (see Smith, 1980; and Smith and Ward, 1981). In many applications equation (14) is modified to include a discoverability parameter (*p*). The parameter is included as the power to which S_j and S_k are raised. When $p = 1$, the discovery probabilities are as expressed in equation (14). When $p > 1$, the discovery probabilities rise for large S_j and fall for smaller S_j.

Typically, however, one does not have knowledge of the original deposition of the fields and it becomes necessary to evaluate the likelihood function, given an observed discovery sequence, at alternative points in the space (n_1 ...n_J), to estimate the original deposition which will maximize the likelihood of the observed discovery sequence having occurred. Once the original depositions have been estimated, predictive probability distributions can be derived by simulating the exploratory process, with the aid of the discovery postulates given above, and observing the relative frequency with which the outcomes occur in a sufficiently large number of trials.

The modelling framework has been applied by Barouch and Kaufman (1976), Smith (1980), O'Carroll and Smith (1980) and Smith and Ward (1981) to estimating the size of the underlying resource base using the historical discovery record. As Smith (1980) points out, ."..application of the model provides direct estimates of the physical returns to continued exploratory activity. Supplemented by estimates of exploratory costs and other economic parameters, the results may be used to infer the economic returns to exploration, and to identify future levels of exploration, discovery, and prediction that are likely to be pursued by the oil industry."

Smith (1980), O'Carroll and Smith (1980) and Smith and Ward (1981), though aware of the concept of a play as a group of similar geological configurations generated by a series of common geologic events, apply their respective models to data sets for the North Sea as a whole and the North Sea Northern Petroleum Province. Such aggregate data sets consist of observations belonging to several natural populations (plays) and clearly violate the single natural population assumption that underpins the probabilistic modelling approach. Furthermore, the models are insensitive to the assumptions made about the form of the parent population of discoveries. The imposition of lognormal and weibull restrictions on the parent population had a "minimal impact on the estimated deposition,"(Smith and Ward, 1981 p.406). Davis and Chang (1989), on the other hand, have stated that "assumptions about the shape of the left side of the pool-size distribution curve are critical to forecasts of the quantity of oil that might be discovered and produced economically from mature petroleum provinces in the United States." The predictive insensitivity of these models appears in stark contrast to the debate in the literature about the frequency-size distribution of oil and gas pools.

In applying the model to the North Sea, several ad hoc adjustments and modelling assumptions were employed. All found it necessary to re-sequence the Statfjord discovery because "it is widely believed that the industry had desired to drill the structure much earlier, at approximately the same time as the Brent field, and was only prevented from doing so by the reluctance of the Norwegian government." Finally, there is no rationale provided for the selection of the discoverability parameter, (*p*). Arbitrary choices are used. The choice of *p*, however, will hold significant implications for model predictions and some algorithm for its choice must be given before any confidence can be placed in model predictions.

Finally, to make accurate predictions of the underlying resource base, the model presumes no economic truncation, i.e. the tendency of firms to report small, currently non-commercial discoveries as dry holes. The phenomenon of truncation, and the problems it posed for resource estimation, were recognized as early as 1958 by Arps and Roberts. Since then there has been a small but growing literature addressing the implications of truncation for resource base estimation. Drew, Attanasi and Schuenemeyer (1988) state that as a result of economic truncation one "should not be confident about inferring the form and specific parameters of the parent field size distribution from the observed distribution." This, however, is precisely what the modelling approach of Barouch and Kaufman (1976), Smith (1980), O'Carroll and Smith (1980) and Smith and Ward (1981) attempts to do. Power and Jewkes (in press) attempt to avoid the problems associated with economic truncation by using exogenously produced geologic estimates of the frequency-size distribution. An exogenous geologically controlled approach also avoids having the modeller make assumptions about the form of the parent frequency-size distribution and allows the model to be effectively used to forecast future discovery rates and marginal finding costs (Power and Fuller, 1992).

Kaufman, Balcer and Kruyt (1975) developed a probabilistic discovery model to reflect observed geologic facts based on the assumptions "that describe the manner in which exploration technology and observed statistical regularities of the size of pools interact to generate discoveries." Predicated on the assumption that the underlying distribution of pool sizes was lognormal, the model used postulates (1) and (2) above and information on the sequence of historical discoveries to make inferences about the parameters describing the parent population of pools and the frequency-size distribution of the undiscovered resource base. Lee and Wang (1983a, 1983b, and 1985) have refined and improved this modelling approach. The Lee and Wang approach attempts to estimate the frequency-size distribution of the resource base without reference to its economic limits. Estimating the pool-size distribution without reference to its economic limits allows the resource analyst to distinguish between changes in reserve base estimates induced by economic influences and changes in resource base estimates induced by changes in geologic knowledge. While the distinction may appear subtle, it is nonetheless an important conceptual refinement. Without good estimates of the resource base, even the most sophisticated of economic predictions of the price induced supply response runs the risk of over- or underestimating recoverable potential because of changes in geologic knowledge that are independent of changes in price.

Lee and Wang (1985) and Coustau (1988) employ the lognormal distribution in their respective discussions and applications of the model. While lognormality is assumed, it is not a key feature of the modelling approach. Any regular parametric family of distributions could as easily be employed. The point is important because there is not unanimous support for the lognormal distribution in the literature. See, for example, Schuenemeyer and Drew (1983), Baker et al. (1984), Houghton (1988), and Schuenemeyer and Drew (1991). Finally, the Lee and Wang approach allows not only estimation of the undiscovered pool-size distribution but also incorporates a useful feedback mechanism allowing analysts to match predictions to known discoveries. The geologic control achieved through the matching process significantly enhances the reliability of parameter estimates.

Meisner and Demirmen (1981) and Forman and Hinde (1985) discuss the development of the creaming methods. The approach recognizes that both the discovery rate and field sizes decline as exploration advances. Trends in both are estimated from the available data and the results used to produce predictive probability distributions of remaining discoveries. Application of the modelling approach to the North Sea and in Australia have demonstrated the forecasting potential of the method. Statistical control, however, requires a sufficiently detailed data set and, as a result, limits the application of the method to mature petroleum provinces.

A popular method of obtaining geologically based estimates of the remaining resource base and the field-size distribution is simulation. The method is particularly popular with large companies having the resources to meet the data, skills and computing requirements of implementing such a system. Whereas that may be its principal weakness, its principal strength lies in the recognition that exact answers are not attainable.

The simulation method has proven to be particularly suited to the judgemental geologic prediction methods of estimating total discoverable reserves. It seeks to define, from a series of estimated probability distributions of field characteristics, the likely distribution of possible field discovery sizes. When coupled with a distribution of expected economic parameters, the method can also define the likely distribution of the expected net present value of the undiscovered resource base.

The factors which control the existence and size of pools in the undiscovered resource base can be divided into two groups: those factors which control the physical occurrence of the resource (geologic factors) and those factors which control its recoverability (technical factors). The geological factors may be subdivided into play, prospect and size specific groupings. Play specific factors include characteristics common to a homogeneous grouping of reservoir rock and trap type that control the occurrence of hydrocarbons within a play. Prospect specific factors consist of geologic characteristics common to all prospects within the play and determine whether an individual prospect will contain hydrocarbons. They include the existence of a trapping mechanism, minimum effective porosity and the probability of hydrocarbon accumulation. Size specific factors consist of geologic characteristics unique to the prospect and determine the volume of hydrocarbons contained in the prospect. They include: prospect areal extent, net pay thickness and porosity.

The algorithm for combining the geologic factor distributions to produce the frequency-size distribution of the undiscovered resource base is depicted in Figure 2. It consists of a series of independent and repeated samplings from each of the defined variable distributions. The size specific factors are sampled first and multiplied to produce a conditional discovery size. The prospect specific factors are then multiplied together to estimate the probability that a randomly sampled prospect contains hydrocarbons. A sampling is then completed from a uniform distribution and, if the sampled figure is less than or equal to the estimated probability of the prospect containing hydrocarbons, the prospect is treated as a discovery and the resulting point estimate is stored. When a sufficiently large number of samplings have been completed, a frequency-size distribution defining the expected sizes of fields constituting the undiscovered resource base is constructed.

Because discoveries within an area of interest are neither homogeneous in nature nor completely recoverable, technical factors are

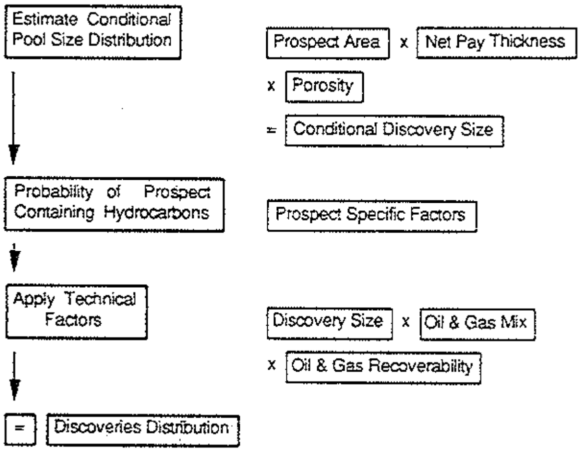

Figure 2: The Frequency-size Distribution of an Undiscovered Resource Base Algorithm

Depiction of an algorithm for combining the physical and technical factors influencing the occurrence of hydrocarbons. Typically a conditional prospect size is estimated using a volumetric approach. Prospect-specific factors are used to determine the likelihood of it actually containing hydrocarbons. For those containing hydrocarbons, a series of technical factors are applied to the *in situ* estimates to determine the probable recoverable volumes contained in the discovery.

applied to the initial volume-in-place estimates, defined above, before the level of recoverable reserves is estimated. The technical factors include an oil-gas mix factor and oil and gas recoverability factors. The oil-gas mix factor is intended to split the resource volume into its component oil and gas elements and to allow for a more detailed recovery rate analysis. The recoverability factors describe the current state of recovery technology and allow recognition of the problems of recovering the resource from specific formations under specific operating conditions.

Both the geological and technical factors can be obtained either by direct estimation or analogy. They are often, though not exclusively, obtained by polling experts, using a Delphi approach, on the occurrence and recoverability of oil and gas in a given area. Typically, the experts are asked to define subjective probability distributions for each of the required factors. One of the advantages of the simulation method is that the uncertainties of the economic evaluation process can easily be appended to the basic analysis. The distribution of likely drilling costs, operating costs and future resource prices can be estimated in a manner similar to that of other model parameters. The new distributions are then combined in the algorithm to produce estimates of exploration profitability. The analysis can also be expanded to take account of the number of producing wells, well productivity, well decline rates, future price movements and operating costs to arrive at a distribution of the expected net present value of the undiscovered resource base.

The chief advantage of simulation analysis lies in the manner in which it breaks the undiscovered resource appraisal question into small component estimations. It is important to estimate the parameters influencing the occurrence of resource deposits separately, rather than directly, because the factors which control the occurrence and recovery of oil and gas deposits are complex. Unfortunately, the method does not avoid the use of subjective probabilities. Nevertheless, it focuses attention on the factors controlling the incidence of oil and gas discovery and recovery, and combines the derived subjective probabilities in a well defined objective fashion. Roadifer (1975) has stated that the " ... estimates resulting from this kind of rigorous analysis are valuable for private company selection of exploration ventures. Because the estimates are presented as probability distributions, they provide an evaluation of the chances there may be large volumes of hydrocarbons as well as near certainty of estimated small volumes."

Conclusions

The methods available for quantifying expectations about the size of the resource base are more numerous than the review above indicates. A complete description of the available methods would require more space than is available in this paper. Additional description of methods would not change the basic classification of methods into the groupings of judgemental prediction, extrapolative, discovery process, econometric and probabilistic techniques. The judgemental prediction methods include the simple geologic analogue, areal and volumetric techniques. Extrapolative approaches typically include logistic or Gompertz curves. The discovery process models include considerations of the phenomenon of resource exhaustion by allowing for the fact that the largest fields tend to be discovered early in the exploration process. The econometric methods are too rich and varied to discuss each in detail. Accordingly, the brief discussion of their basic form offered above must suffice. Finally, there are the probabilistic methods. These consist chiefly of the sampling without replacement models initially developed by Barouch and Kaufman (1976) and the large variety of simulation based models. In many instances the simulation methods employ other modelling techniques at their core. Their distinguishing feature, then, is that each specifically accounts for the uncertainties inherent in exploration and development with statistical sampling routines.

The question that remains is which, if any, of the methods is "best"? The judgemental prediction methods make specific reference to the fundamentals of petroleum generation. Though closely tied to the antecedent geological and geophysical profiling of a region, they tend to require information not easily obtained in the absence of actual drilling. Furthermore, they produce only crude estimates of the resource base. The simple extrapolative methods provide a mechanical means of projecting the ultimate size of the resource base using cumulative historical data. Though simple to apply, they are devoid of theoretical underpinnings and must be dismissed by those seeking to explain what factors influence the ultimate size and recoverability of the resource base. Both methods will continue to have their uses and proponents. However, neither can be effectively employed by firms or policy makers to meet their respective information needs.

The most sophisticated approaches are the geologically based (probabilistic and discovery process) and the econometric approaches. Each draws specifically from the theoretical basis of either geology or economics to formulate models highlighting fundamental points about the exploration and discovery process. The geologically based models draw on observations concerning the discovery and deposition of the resource — the tendency of largest fields to be discovered early in the exploration process and the preponderance of smaller fields — to estimate the probable size of the resource base based on past discoveries. The econometric models rely on behavioral relationships describing the markets for and the inputs required to exploit the resource in order to predict the evolution of discoveries given the specifics of future prices and effort.

A useful framework for distinguishing between the various approaches is the classification of mineral reserves and resources system approved by the United States Geological Survey. The system is depicted in Figure 3. Reserves are sensitive to changes in both price and technology and may be added or reclassified as a result of either exploration or changes in the economic environment. Accordingly, the rectangle describing reserves may be expanded, or contracted, by movements in the vertical axis. Improvements in the economic environment thus transform reserves from the marginal or non-economic to economic categories. The rectangle describing reserves may also be expanded by movements in the horizontal axis. These are brought about by exploration, whereby the results of exploration transform resources into reserves.

Resources are a geologic concept and estimate the natural endowment of hydrocarbons. Their existence is not influenced by economics or technology. The fact that a hydrocarbon pool is uneconomic or technically difficult to produce from does not make it disappear. Reserves, on the other hand, are a hybrid geologic-economic concept. From the geologist's point of view, reserves are a sub-group of resources categorized with respect to knowledge of their existence. From an economist's point of view, reserves are a sub-group of resources categorized with respect to their

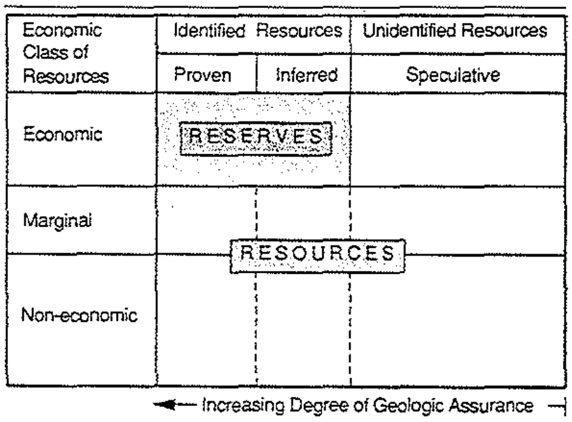

Figure 3: The Classification of Mineral Reserves and Resources

Classification scheme of the mineral reserves and resources approved by the United States Geological Survey. Reserves are sensitive to changes in both price and technology and may be added (or subtracted) by changes in the economic environment or added as a result of the exploration process. However, the resource base remains unchanged.

producibility. As prices and technology change, reserves will change.

The dichotomy between the two approaches lies in their respective treatment of (1) price and the physical factors controlling the incidence of petroleum occurrence and (2) the aim of the methodology. The econometric models take price as paramount and include little consideration of the resource's physical characteristics, principally because they aim at forecasting the discovery rate as affected by variations in price and effort. Estimates of the resource base are produced largely as a consequence of allowing price or effort to rise to infinity. And it is here that econometric methods are weakest. Without an accompanying theory of resource exhaustion, the use of past average discovery sizes as a determining variable by econometric models is quite incorrect, given the clearly established pattern of large fields being discovered early in the exploration process. On the other hand, the geologic based models relegate price to the realm of production-related problems and build their equations around the established facts of petroleum occurrence. That is because the geologic based models aim specifically at estimating the size of the undiscovered resource base. Discovery rate forecasts are produced more as a by-product of resource base estimation and are invariably more closely related to the physical phenomenon of resource depletion than short-term variations in market forces. The phenomenon of economic truncation reported by Arps and Roberts (1958), Schuenemeyer and Drew (1983), and Attanasi and Drew (1985) indicate, however, that price has a role to play in the declaration of resource deposits as economic or sub-economic. Failure to account appropriately for the role of price in determining the portion of the resource base that is commercially usable weakens the strictly geologic approach to resource estimation. However, the use of information on the dwindling size and number of potential discoveries in the geologic-based models yields insights into the petroleum exploration and discovery process denied econometricians.

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