_____

A GAME THEORETICAL ANALYSIS OF THE DESIGN OPTIONS OF THE REAL-TIME ELECTRICITY MARKET

HAIKEL KHALFALLAH
University Pierre-Mendès-France, France

VINCENT RIOUS
Microeconomix, France

ABSTRACT

In this paper we study the economic consequences of two real-time electricity market designs (with or without penalties) taking into account the opportunistic behaviors of market players. We implement a two-stage dynamic model to consider the interaction between the forward market and the real-time market where market players compete in a Nash manner and rely on supply/demand function oligopoly competition. Dynamic programming is used to deal with the stochastic environment of the market and the mixed complementarity problem is employed to find a solution to the game. Numerical examples are presented to illustrate how the optimal competitor's strategies could change according to the adoption or no adoption of a balancing mechanism and to the level of the penalty imposed on imbalances, regarding a variety of producers' cost structures. The main finding of this study is that implementing balancing mechanisms would increase forward contracts while raising electricity prices. Moreover, possible use of market power would not be reduced when imbalances are penalized.

Keywords
Electricity markets, balancing mechanisms, supply function equilibrium, mixed complementarity problem

Corresponding Author:
Haikel Khalfallah, PACTE-EDDEN, UPMF, BP 47, 38040 Grenoble CEDEX 9, France
Email: haikel.khalfallah@upmf-grenoble.fr

1. INTRODUCTION

Electricity requires a constant physical balancing between generation and load. The forward markets, where market players, mainly producers and resellers, schedule their supply and demand before the operating day or some hours before the delivery time, are not sufficient to ensure real-time balancing. The system operator is then in charge of the real-time balance between generation and load by relying on a balancing arrangement. All forward market's participants have to fulfill their commitment and reconcile any differences between the schedule in the forward market and the real-time load. Even if its financial weight is very small compared to those of the forward markets and the forward contracts, the real-time market design is essential for the efficiency of the power markets as a whole particularly due to the electricity industry's characteristics such as non-storability and load uncertainty.

There exist two ways to balance system in real-time: the first one uses the market price to value real-time energy. Market participants then exchange energy at market price, either participating in creating the system imbalance or helping to relieve it. It is called *"real-time market"*[1]. The second arrangement penalizes the imbalance. A penalty is then added to the marginal energy price and paid by market participants that don't balance their schedules *ex ante* in the forward markets. This design is called "*balancing mechanism*"[2]. The basic idea behind the balancing mechanism is to reduce incentives for market participants to voluntarily schedule imbalances and to push them to improve generation forecasting. The main focus of this paper is to compare the efficiency of the two real-time arrangements and their economic consequences when the market participants can exert market power.

In the literature, there exist some papers having focused on the interaction of the two-settlement markets via analytical and computational methods. For instance, Allaz (1992) shows that the implementation of a forward market could guarantee a more competitive spot market, yielding specially decreased spot prices. Kamalt and Oren (2004) extend Allaz (1992)'s study by including two node networks and uncertainty on transmission capacity in the spot market. They point out that generation firms have a great incentive to engage in forward market, which leads to increase social surplus.

There are, however, few works that model the economic consequences of applying a "*balancing mechanism*" or a "*real-time market*" in a two-settlement market. To our knowledge, Saguan and Glachant (2005) is the unique study that focuses on this issue. They apply a simple two-stage market model with perfect competition, with risk-averse market participants and uncertainty on demand in the spot market. Their results show that when a balancing mechanism is present, forward prices are distorted and it could lead to productive inefficiency especially when some generators are inflexible. Our paper extends the work of Saguan and Glachant (2005). First we suppose an imperfect competition in the two markets instead of perfect competition. We then apply the Nash theory of oligopolistic behavior. We

---

[1]For instance, this configuration is close to the arrangements employed in German and Dutch electricity markets. Marginal pricing is also the principle applied in the real time market in the liberalized power systems in the USA.

[2]For instance, this design is currently implemented in France and Belgium.

formulate a two-stage dynamic model to characterize market participants' decisions: on the one hand producers and resellers' schedules in the forward market and on the other hand their final production and consumption in the real-time market. Second we extend the model of Saguan and Glachant (2005) by assuming that market players are price makers. Strategic behaviors of market players are modeled through specific supply and demand functions that determine electricity prices formations in the markets. This extension would help to study market power opportunities and the extent to which the two balancing designs are able to alleviate it[3]. Studying the consequences of introducing balancing arrangements when market players can exert market power and evolving in an imperfect competition environment is the economic contribution of this paper.

In the energy area, imperfect competition is generally modelled by supposing Cournot competition. Cournot models have the advantage of being easier to formulate and to solve while they are more sensitive to the parameterization, in particular of demand elasticity[4]. Our work relies however on supply function equilibrium models. Even though supply function equilibrium models have the drawbacks of being more difficult to formulate and to solve, they require far less parameters and are less sensitive to assumptions[5]. Besides, supply function competition is more likely to correspond to the real behavior of market participants in both forward and spot electricity markets. Indeed, in real world market actor's bid is a set of price-quantity couples than either a fixed quantity or a fixed price, which could be approximated by a supply function relating his quantity to his price. It is so assumed that each producer offers a supply function that expresses the optimal relationship between his production and the minimum price he then accepts. Similarly, each reseller offers a demand function expressing the relationship between his demand and the maximal price he is willing to pay.

Our game is a two-stage dynamic game. Market participants, who are risk-adverse, schedule production and demand against each other in the forward market (first stage), knowing how they will play against each other when also participating in the spot market (second stage). Owing to the presence of uncertainty in this latter market, a closed-loop solution[6], based on stochastic dynamic programming approach[7], is used to represent the interaction between all market players' decisions. More precisely, we suppose that market players' decisions in the forward market and the real-time market are not taken at the same time. Forward decisions are made in the first stage and real-time decisions in the second stage. We proceed by backward induction to find the solution of the game. The resolution starts from the second

---

[3]The case of inflexible generators has been already studied in the paper of Saguan and Glachant (2005). Knowing that such configuration would only bring complexity without modifying our conclusion, we assume that all the producers are flexible in real time.

[4]See Pineau and Murto (2003), Von der Fehr and Harbord (1997), Murphy and Smeers (2005), Chuang and al. (2003) and Khalfallah (2011) for different studies relying on Cournot models applied to the liberalized electricity systems.

[5]See Day (2002), Anderson and Xu. (2006) and Willems et al. (2009) for a broad literature review on the use of supply function equilibrium.

[6]A good comparison between the open-loop and closed-loop solutions in electricity markets is proposed in Murphy and Smeers (2005).

[7]Dynamic programming is an approach developed to solve sequential, or multi-stage, decision problems. It divides the problem to be solved into a number of sub-problems and then solves each in such a way that the overall solution is optimal to the original problem (Bertsekas 2000).

_____

(last) stage by determining optimal quantity and price in the real-time market, given demand uncertainty scenarios and constrained by market players' forward commitments. In mathematical words, this means that we calculate optimal reaction functions of second stage players' strategies, function of first stage strategies. At the first stage players determine their commitments in the forward market by taking into consideration how they will react in the real-time market.

Since market players' decisions for a given stage are taken simultaneously and under Nash game assumptions, we rely on the mixed complementarity problem (MCP) technique to calculate the equilibrium at each stage: optimal reaction functions in the second stage and optimal forward quantities and prices in the first stage. This technique is used in a considerable volume of literature that deals with imperfect competition, mainly to describe the production-investment problem (Ventoza et al. 2002, Murphy and Smeers (2005) and Pineau and Murto (2003)). The key assumption that permits to reformulate players' optimization problem as a MCP problem is that each player decides in a Nash manner. Consequently, under this approach, the Nash equilibrium defines the set of prices and quantities that simultaneously satisfies the first order optimality conditions of all players. The resulting MCP model can be solved taking advantage of its complementarity structure[8].

The paper contributes to the literature by relying on a combined use of MCP and closed-loop information structure to simulate interactions between risk-adverse market players who compete under supply function equilibrium competition and in a stochastic environment. Even though Khalfallah (2011) used the above mathematical approaches to model the long-term effect of investment incentive mechanisms for generation capacity; and to characterize the interactions between actors' decisions in an uncertain environment, his work supposes however risk-neutral market players competing under a Cournot model.

Our paper is organized as follow. First we describe the basic assumptions of the model. Second we formulate the problem. Third, considering that the overall game cannot be solved analytically because the assumption of continuity is not always verified, we illustrate the model with numerical examples and we analyze them to evaluate whether the implementation of a balancing mechanism helps to limit the exertion of market power at the forward and real-time stages. From our results, we conclude with recommendations about the design of the balancing arrangements.

2. MODELING ASSUMPTIONS

Our two-settlement system is implemented as follow. In the forward market, producers and the reseller schedule their production and load before the operating day. All the market participants face uncertainty with regard to the demand level (D_w) when participating in the forward market. In the balancing step in real-time, the demand is realized and market participants must balance their schedules accordingly. Here, only producers are active players, meaning that they compete to decide their adjustment production level. Contrarily, the reseller is a passive player. He faces an obligation in real-time to serve a demand completely price-inelastic. He must therefore purchase or sell any difference between his forward quantity and the

_____

[8]More details on this technique are given in Appendix B.

realized one, in order to satisfy his specific consumers demand.

2.1   MARKET PARTICIPANTS

There are three types of actors to consider with different interests:
- Producers: We assume there are two producers (c) who schedule production in the forward market and adjust it in real-time (following the system operator's order – see below) to ensure the balance between generation and load.
- Reseller: We consider a unique reseller supplying an inelastic demand at a contracted fixed price of P^r. He purchases electricity in the forward market in a variable amount, depending on the producers' offer and given his demand function. He is also obliged to balance demand from his end consumers in real-time whatever the price of electricity then[9].
- System operator: We assume that a system operator is in charge of balancing the system and to clear the balancing arrangement in real-time.

2.2   MODELING UNCERTAINTY

In the forward market stage, the demand level is uncertain. It will be set later on in real-time. We assume that the stochastic demand can take four possible states with a uniform probability: going from low demand to high demand.

We assume that market participants have a risk-averse behavior in the forward market. We model it with a utility function for the market participants comprising not only their expected surplus but also a penalty related to uncertainty on their surplus (see section 3 for details).

2.3   THE FORWARD MARKET

In the forward market, market participants offer quantity and price via a specific function. Each producer offers a supply function that expresses the relationship between the quantity he can produce and the minimum price he accept to earn. Similarly, the reseller offers a demand function expressing the relationship between his demand and the maximal price he is willing to pay. The market is cleared at the intersection between the aggregate offer function and the demand function. The forward equilibrium price is then determined.

2.4   THE REAL-TIME STEP AND THE BALANCING ARRANGEMENT

We distinguish two kinds of balancing arrangements. The first one is a basic market and is so called *real-time market*. The market participants adjust their forward schedules once they know the real demand level and contract energy in real-time at the marginal electricity price without any additional penalties applied to imbalances. The principle of marginal pricing is hence applied to the balancing arrangements in some European countries like Germany and the Netherlands (see Rivero et al., 2011) and in most of the liberalized electricity systems in the USA through nodal pricing (Neuhoff et al. 2011).

The second kind of balancing arrangement has the peculiarity to apply a penalty on the imbalance for resellers beyond the marginal price of electricity in real-time. This arrangement is called balancing mechanism. The producers are otherwise paid

---

[9]Compared to reality, our definition of reseller encompasses both the role of the supplier and the role of the balancing responsible party.

at the marginal electricity price. The argument grounding this design is that the reseller has then a smaller incentive to rely on the balancing arrangements to supply the load of his final consumers. In other words, a part of the balancing responsibility is transferred from the TSO to the reseller. The TSO should then encounter a smaller number of events where the reseller imbalances endanger the system reliability. Balancing mechanisms are applied in France and Belgium (see Rivero et al., 2011). The penalty is generally calculated as a function of two main parameters, first the type of imbalance for the reseller (positive, i.e. an excess of generation compared to load or negative, i.e. a deficit of generation compared to load), and second its sign compared to the sign of the system imbalance (i.e. does the reseller imbalance help to rebalance the system or does it participate in the system imbalance?). The penalty can be computed in different ways, in particular relying on marginal price vs. an average offer price[10]. To simplify our model, the penalty is computed from the marginal price: the negative (or positive) imbalances prices are explicitly computed multiplying (or dividing) by a constant (1+k), the marginal price of electricity in the real-time market[11].

3. THE MODELS

We develop a two-stage dynamic model to simulate market participant decisions in both the forward market and real-time. The objective of each player is to maximize his total utility, function of his profit from participating in the two stages.

For each producer, the objective function to be maximized is described as follows:

$$\max_{q_{c,RT}, q_{c,F}, P_{c,TR}, P_{c,F}} \quad U_c = \pi_{c,F} + E_w\big(U(\pi_{c,RT,w})\big) \tag{1}$$

Subject to,

$$\sum q_{c,RT,w} = D_w - \sum q_{c,F} \tag{2}$$

$$\sum q_{c,F} = x_{r,F} \tag{3}$$

$$q_{c,F} \geq 0 \tag{4}$$

The notation is defined as follows:

---

[10]The penalty in the balancing mechanism can either be applied to the marginal price of electricity in real time or to the average price of offers required to compensate imbalances. In this later situation, the distortion on imbalance price is worsened since it does not rely on marginal pricing at all.

[11]Since there is only one reseller in our model, his imbalance corresponds to the system imbalance too.

$c = 1,2$	The producers
w	Scenarios of the real-time demand
$q_{c,F}$	Quantity sold by producer c in forward market
$q_{c,RT}$	Additional quantity sold by producer c in real-time (positive or negative)
$x_{r,F}$	Quantity bought by reseller r in forward market
$P_{c,F}$	Producer c's supply function in the forward market
$P_{c,RT}$	Producer c's supply function in real-time
D_w	Demand level in scenario w
U_c	Utility function of producer c
$\pi_{c,F}$	Producer c's revenue in the forward market
$\pi_{c,RT,w}$	Producer c's revenue in real-time with demand D_w
$E_w(U(\pi_{c,RT,w}))$	Expected utility function of producer c in the real-time market

The constraints specific to producers' supply function are omitted here and will be presented later. The objective of each producer is to maximize his utility function taking into account his risk-averse behavior with regards uncertainty on the real-time demand when scheduling his production in the forward market. So, the utility function (1) corresponds to the sum of his revenue from the forward market and his expected utility from buying or selling energy in the coming real-time step.

The reseller faces an uncertain demand in real-time. He must then buy energy in the forward market while demand is uncertain at that time and will be eventually observed only in real-time. We introduce a random variable w that indicates possible demand realization in real-time and corresponds to a finite set of scenarios. Put differently, this random variable has a discrete distribution function.

We assume a risk-averse behavior of market participants. This is formalized by undervaluing an uncertain profit. The expected utility of profit function in real-time then takes the following form:

$E_w(U(\pi_{c,RT,w})) = E_w(\pi_{c,RT,w}) - \frac{A_c}{2} Var(\pi_{c,RT,w})$[12]. A_c is the risk aversion parameter of producer c and, $E_w(\pi_{c,RT,w})$ and $Var(\pi_{c,RT,w})$ are respectively the expected value and the variance of the producer's profit in real-time.

In the model (1)-(4), each producer decides his optimal quantities to be offered in the forward market and in real-time, $q_{c,F}$ and $q_{c,RT}$ respectively, and the corresponding supply function in both steps, $P_{c,F}$ and $P_{c,RT}$ respectively. Constraint (2) means that total production from forward scheduling ($\sum q_{c,F}$) and real-time adjustment ($\sum q_{c,RT,w}$) is equal to the real-time realized demand (D_w). Constraint (3) ensures that the total quantity offered by producers in the forward market equals the total quantity bought by the reseller ($x_{r,F}$).

On the other hand, the reseller purchases electricity from the forward market and in real-time and sell it to customers at a contracted fixed price. However, he is an active player only in the forward market where he offers a demand function expressing the maximal price he is willing to pay for different quantities. In real-time, he buys or sells the difference between the realized demand and his previous

---

[12]This kind of utility function was used for exemple in Bessembinder-Lemmon (2000), Saguan and Glachant (2005) and Saguan (2007)

forward purchases. He determines his optimal offer in the forward market by maximizing his utility function, taking into account his risk-aversion to demand uncertainty. The reseller's optimization problem is described as follow:

$$\max_{x_{r,F}, P_{r,F}} \quad U_r = \pi_{r,F} + E_w\big(U(\pi_{rF,w})\big) \qquad (5)$$

Subject to,

$$\sum q_{c,F} = x_{r,F} \qquad\qquad\qquad (6)$$

$$x_{r,F} \geq 0 \qquad\qquad\qquad (7)$$

The notation is defined as follows:

r	Reseller
$P_{c,F}$	Reseller's demand function in the forward market
U_r	Utility function of the reseller
$\pi_{r,F}$	Reseller's revenue in the forward market
$E_w\big(U(\pi_{r,RT,w})\big)$	Expected utility function of the reseller in the real-time market

3.1   SOLVING THE MODELS

Our game is a two-stage dynamic game where Nash competition takes place in two steps. First, in the forward market, producers and resellers simultaneously decide quantities and prices – in the form of supply and demand functions. The second stage is in real-time where only producers are in competition and determine their optimal quantities in order to satisfy the realized demand level, taking into account previous forward contracts and their optimal supply functions. Second stage decisions are then constrained by first stage's ones.

Given our game configuration, we use the closed loop information structure to solve the model. The solution is obtained by backward induction. Knowing that real-time market decisions are made on the basis of the observed forward contracts, we first solve the real-time stage parameterized by the forward variables. And in a second step, we solve the forward stage integrating the solutions from the real-time one[13].

At last, a stochastic dynamic programming method is used to solve our model in numerical examples. The essence of dynamic programming is Bellman's principle of optimality[14].

In the following sections, we calculate the Nash equilibrium associated with each

---

[13]Except for the difficulty generally encountered in the use of this technique, it is sub-game perfect because the associated strategies are Nash equilibrium at each stage of the game, even if there has been a deviation from the equilibrium strategy in an earlier sub-game, contrary to the open-loop information structure.

[14]Bellman's principle of optimality states that: 'An optimal policy has the property that, whatever the initial action, the remaining choices constitute an optimal policy with respect to the sub-problem starting at the state that result from the initial action'.

_____

stage of the game and, using the backward induction method, we start from the last decision and end up with the first one.

3.2   THE BALANCING STAGE

In real-time, the system operator is responsible to balance the system. He can so order producers to increase or decrease their production level in order to compensate the difference between the realized demand and previous commitments in the forward market. Producers enter in Nash competition in order to determine the optimal dispatch of the adjusted quantity and the balancing price.

3.2.1 The Producers' problem at the balancing stage:

To solve the producer's problem at the balancing stage, we have to distinguish between two situations depending on the sign of imbalance. In case of negative imbalance, i.e. when the realized demand exceeds the total forward quantity, producers should offer a positive quantity and balance the system. However, when the imbalance is positive, they should "buy" back energy, i.e. decrease their production level.

<u>*When imbalance is negative*</u>

Supposing that forward positions and price are already determined, producers determine the additional quantity to be sold in real-time and the corresponding market price considering only the balancing stage. Producer *c*'s problem then is formulated as follows:

$$\max_{a_{c,RT,w},q_{c,RT,w}} \qquad P_{RT,w} \cdot q_{c,RT,w} - CT_{c,RT,w}\left(q_{c,F}^*, q_{c,RT,w}\right) \qquad (8)$$

Subject to,

$$\sum q_{c,RT,w} = D_w - \sum q_{c,F}^* \qquad\qquad\qquad (9)$$

$$q_{c,RT,w} \geq 0 \qquad\qquad\qquad\qquad (10)$$

$$a_{min} \leq a_{c,RT,w} \leq a_{max} \qquad\qquad\qquad (11)$$

Where,

$q_{c,F}^*$	Quantity sold by producer *c* in the forward market
$CT_{c,RT,w}(.) = m_c \cdot Q_{c,w} + n_c \cdot Q_{c,w}^2$	Quadratic variable cost function of producer *c*
$P_{TR,w}$	Energy price in real-time calculated by aggregating producers' supply function
$a_{c,RT,w}$	Specific variable of producer *c*'s supply function (see below)

We assume that each firm chooses his supply function in order to maximize his profit in the balancing stage. It stands for the energy price $P_{c,RT,w}$ requested by the producer as a function of the additional production planned from the balancing stage $q_{c,RT,w}$. It is a twice differentiable, monotonically non-descending function of quantity and takes the following form:

$$P_{c,RT,w} = a_{c,RT,w} + b_c . q_{c,RT,w} \qquad (12)$$

Under perfect competition conditions, producers bid their marginal cost in the real-time market. Nevertheless, in our model the firms play a non-cooperative oligopoly game. We suppose that producers' supply functions could be different from their marginal costs, depending on their strategic behaviors in a given market scenario and market power exertion opportunities. Mathematically, this is formulated by setting the producer's supply function (12) as a function of his marginal cost function. Without any market power, a producer's supply function corresponds to his marginal cost, i.e. parameter b_c in the supply function equals the slope of the marginal cost function $(2. n_c)$ and $a_{c,RT}$ is equal to the parameter m_c of the producer's cost function. When market players behave strategically, they could act on the the variable $a_{c,RT}$ which is supposed to be a free and bounded variable[15]. Producers can adjust it in order to increase or decrease the equilibrium energy price while the parameter b_c in the supply function is always equal to the slope of the marginal cost function $(2. n_c)$[16]. The electricity price is then found by aggregation of individual supply functions of producers, as described in appendix A.

Each firm maximizes simultaneously his profit (8) under constraints (9), (10) and (11). The Nash equilibrium is unique since the cost function is strictly convex and continuously differentiable, and the revenue function is concave. The solution is found by setting and grouping together all producers' first order optimality conditions. A mixed complementarity problem is then formed (see Appendix B for more details). After solving the problem, we find the producer's optimal additional quantity in real-time $q^*_{c,RT,w}$, function of the quantity sold in the forward market $q^*_{c,F}$ and of the optimal level $a^*_{c,RT,w}$ of the variable $a_{c,RT,w}$ which illustrates his specific supply function:

Lemma 1. If producers are symmetric (i.e. the same cost structure: parameters m_c and n_c are equal for both producers), they will contract the same quantity in the forward market. As the imbalance is negative, forward schedule of each producer is equal or less than half of the realized demand level, i.e. $q^*_{c,F} \leq \frac{1}{2}. D_w$. Consequently, for a given realized real-time demand level and for given forward contracts, realtime adjusted production of each producer is:

---

[15]In order to guarantee the existence and the uniqueness of the equilibrium, we suppose that this variable is bounded.

[16]In the studies dealing with market power in electricity market, usually the slope of the supply function is supposed to vary. For our case, to ensure the existence of the solution and for sake of simplicity, the y-intercept of the supply function is supposed to be variable. This assumption does not change the nature of the equilibrium since only the intersection between the aggregated supply function and demand function is important for determining the electricity price et the equilibrium.

$$- \; q_{c,RT,w}^* = \frac{1}{2}.D_w - q_{c,F}^* \tag{13}$$

That means that, when producers are symmetric, the quantity needed to balance the system is symmetrically dispatched among them. As well, each producer will offer its highest supply function in the real-time market where the variable $a_{c,RT,w}$ is at its maximum level:

$$- \; a_{c,RT,w}^* = a_{max} \tag{14}$$

Lemma 2. If producers have asymmetric cost structures, i.e. $m_c \neq m_{c'}$, and for a given realized real-time demand level and for given forward contracts:

$$- \; q_{c,RT,w}^* =$$

$$\begin{cases} D_w - \sum q_{c,F}^* & \text{where producer } c \text{ has the lowest cost} \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

And for both producers:

$$- \; a_{c,RT,w}^* = a_{max} \tag{16}$$

When producers are asymmetric, only the low-cost producer offers the quantity to balance the system.

From Lemma 1 and Lemma 2, we can deduce that each producer offers at the equilibrium his highest supply function in the market when the imbalance is negative. Since the reseller is not an active player in real-time (because he must satisfy an inelastic demand), producers can exert market power by increasing the balancing price at maximum.

When imbalance is positive:

In this case, producers act as energy "buyers". They have to buy back electricity in order to compensate for the difference between their forward sales and the realized demand in order to balance the system. Optimal quantity and equilibrium price are determined by minimizing simultaneously the total producer c's payment, corresponding to the sum of producer's energy payment and production cost. The producer's problem in this case formulates as follow:

$$\min_{a_{c,RT,w}, q_{c,RT,w}} \qquad P_{RT,w}.q_{c,RT,w} + CT_{c,RT,w}\left(q_{c,F}^*, q_{c,RT,w}\right) \tag{17}$$

Subject to,

$$\sum q_{c,RT,w} = \sum q_{c,F}^* - D_w \tag{18}$$

$$q_{c,RT,w} \geq 0 \tag{19}$$

Producers supply functions are calculated similarly to the negative balance case and have the same formulation as in (12). Except that producers determine their optimal offer by minimizing their total payment in the real-time market stage instead of maximizing their profit. Optimal supply function and quantity bought at this stage are calculated as explained in appendix A and in the sub-section above. At the equilibrium:

Lemma 3. If producers are symmetric, as the imbalance is positive, forward schedule of each producer is equal or more than half of the realized demand level, i.e. $q^*_{c,F} \geq \frac{1}{2}.D_w$. Consequently, for a given realized real-time demand and for given forward contracts:

$$- \ q^*_{c,RT,w} = q_{c,F} - \frac{1}{2}D_w \tag{20}$$

When producers are symmetric, the quantity needed to balance the system is symmetrically dispatched. As well, each producer will offer its lowest supply function in the real-time market where the variable $a_{c,RT,w}$ is at its minimum level:

$$- \ a^*_{c,RT,w} = a_{min} \tag{21}$$

Lemma 4. If producers are asymmetric, for a given realized real-time demand level and for given forward contracts:

$$- \ q^*_{c,RT,w} = \begin{cases} \sum q^*_{c,F} - D_w & \text{where producer } c \text{ has the lowest cost} \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

And for the two cases:

$$- \ a^*_{c,RT,w} = a_{min} \tag{23}$$

When producers are asymmetric, only the producer with the highest cost buys the quantity that balances the system in real-time. Moreover, each producer offers at the equilibrium his lowest supply function in the market. Here he exerts market power lowering the price (because they are buyers).

3.2.2. The reseller's problem:
At this stage, the reseller is a passive player. He buys (or sells) the difference between the realized demand level (D_w) and his previous forward purchases ($x_{r,F}$). Therefore quantities bought or sold in the real-time market are:

$$\forall w \ ; \ x_{r,RT,w} = x_{r,F} - D_w \tag{24}$$

The price paid by the reseller in real-time will depend on whether a real-time market or a balancing mechanism is applied. When no penalties are applied for imbalances, the reseller pays (or earns) the real-time marginal price of

electricity $P^*_{RT,w}$. However, when a balancing mechanism is implemented, the price will be different from the real-time market clearing price.

The imbalance price paid by the reseller then depends on the sign of the imbalance and on the degree of penalty k chosen by system operator. We can define so imbalance prices as follows:

$$IP_{RT,w} = \begin{cases} if\ x_{r,RT,w} \leq 0 & P^*_{RT,w}.(1+k)(negative\ imbalance) \\ if\ x_{r,RT,w} > 0 & \frac{P^*_{RT,w}}{(1+k)}\ (positive\ imbalance) \end{cases} \tag{25}$$

Where k is the penalty coefficient ($k \geq 0$). Note that if $k=0$, the imbalance price corresponds to the real-time marginal price of electricity.

3.3   FORWARD MARKET

After calculating optimal players' strategies in the real-time stage, we formulate a stochastic dynamic problem in order to find optimal forward contracts of producers and reseller. This formulation takes the form of stochastic Bellman's equation (equation 26 below). That is to say that, in this stage, market participants offer quantities and prices through their supply and demand functions, knowing how they will optimally react in the coming balancing stage, for each realization of the expected real-time market demand.

Due to uncertainty on the demand level and risk-averse behavior of market participants, each one determines his optimal strategy in the forward market by maximizing his total utility function. The utility function for the producers is as follows:

$$\max_{q_{c,F},a_{c,F}} \quad U_c = P_F.q_{c,F} + E_w\left(U(\pi_{c,RT,w}(q^*_{c,RT,w}, P^*_{RT,w}))\right) \tag{26}$$

Subject to,

$$\sum q^*_{c,RT,w} = D_w - \sum q_{c,F} \tag{27}$$

$$\sum q_{c,F} = x_{r,F} \tag{28}$$

$$q_{c,F} \geq 0 \tag{29}$$

$$a_{c,min} \leq a_{c,F} \leq a_{c,max} \tag{30}$$

Where,

$q_{c,F}$ Quantity sold by producer c in the forward market

P_F Price in the forward market

$a_{c,F}$ Specific variable of producer c's supply function in the forward market

$q^*_{c,RT,w}$ Optimal additional quantity that producer c sells or buys in real-time

$P^*_{RT,w}$ Balancing price paid to producers in real-time

As in the real-time stage, we assume that firms choose their supply functions in order to maximize their total utility functions. The supply function is formulated as follow:

$$P_{c,F} = a_{c,F} + b_c.q_{c,F} \tag{31}$$

Similarly, the reseller maximizes his total utility function defined by:

$$\max_{x_{r,F},a_{r,F}} \quad U_r = -P_F.x_{r,F} + E_w\big(U(\pi_{r,RT,w}(x^*_{r,RT,w}, P^*_{RT,w}))\big) \tag{32}$$

Subject to,

$$\sum x^*_{r,RT,w} = D_w - \sum x_{c,F} \tag{33}$$

$$\sum q_{c,F} = x_{r,F} \tag{34}$$

$$x_{r,F} \geq 0 \tag{35}$$

$$a_{r,min} \leq a_{r,F} \leq a_{r,max} \tag{36}$$

Where,

$x_{r,F}$ Quantity bought by the reseller in the forward market

$a_{r,F}$ Specific variable of the reseller demand function in the forward market

$x^*_{r,RT,w}$ Optimal additional quantity that a reseller sells or buys in real-time at the imbalance price

The reseller is an active player at the forward stage. He offers a demand function expressing the relationship between the quantity he wants to buy and the maximal price he is willing to pay. It is a decreasing function that formulates as follows:

$$P_{r,F} = a_{r,F} - b_r.x_{r,F} \tag{37}$$

As we can see from these two models, producers and the resellers exchange energy at the same energy price P_F. It corresponds to the intersection between the aggregate supply function and the demand function (Figure 1). Producer c determines $a^*_{c,F}$ and $q^*_{c,F}$ and reseller determines $a^*_{r,F}$ and $x^*_{r,F}$ to maximize their individual utility function under the constraint of equilibrium between the forward scheduled production and the forward scheduled demand.

When offering in the forward market, producers and the reseller act principally

on variables $a_{c,F}$ and $a_{r,F}$ respectively. The market is cleared at the intersection between the total supply function and the demand function. The quantity bought by the reseller is equal to the total quantity sold by producers. Graphically:
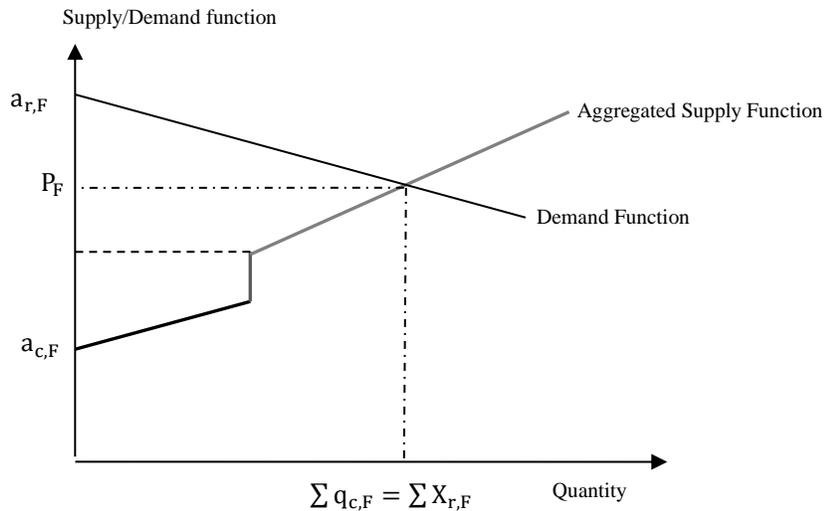

Figure 1. Forward market equilibrium

The expected utility function $E_w(U(.))$ of each market player is not a strictly continuous function, owing to the fact that optimal reaction functions calculated in the real-time stage, in terms of quantities and prices[17], will depend on the real-time imbalance sign and so on the quantities contracted at the forward stage. Consequently, resolving the programs (26-30) and (32-36) by MCP method cannot yield to analytical solutions. We find Nash equilibriums, however, by using an iterative search procedure based on numerical simulations[18]. The solution algorithm executes at two levels. At the first level, we divide the feasibility domain of the decision variables specific to the first stage to different sub-intervals for each of them. All market players' objective functions are simultaneously continuous. For each sub-interval, a Nash sub-equilibrium is calculated. Then at the second level, from a given Nash sub-equilibrium in a given sub-interval, each player re-optimizes his utility function one by one in turn with all other players' actions fixed. The algorithm searches for a Nash equilibrium solution to the overall game which is reached when no player has interest to unilaterally modify his strategies[19].

4. NUMERICAL EXAMPLES

Numerical simulations serve to understand and compare fundamental and qualitative characteristics of the two studied designs for balancing arrangements, namely the real-time market and the balancing mechanism.

---

[17]As defined in section 3.2.

[18]Numerical simulations could be seen as a good alternative to analytical studies when it is impossible to find analytical solutions (Walliser (2005)).

[19]Chuang and al. (2003) have used a similar iterative search procedure to find the Cournot equilibrium of their investment-production model.

These two designs are compared in three scenarios regarding producers' types, i.e. with producers having similar cost structures (symmetric producers) and producers having different cost structures (low asymmetric producers and high asymmetric producers). Producers are symmetric if they have the same cost function formula i.e. m_c and n_c are the same while are asymmetric if m_c parameters differ. We note that parameter n_c is supposed to be identical whatever the scenario.

In the following sections, we first describe the parameters used to compare the two balancing designs. Then, we study the forward and real-time (balancing) prices and exchanged quantities. We also analyze the extent to which market players exert market power in the two designs. At last, we realize a sensitivity analysis, varying the parameters' values to test the robustness of our results.

4.1. PARAMETERS

The parameters of cost functions are estimated from historical data of the French electricity market, and found in DIGEC (1997) and DGEMP (2003)[20].

We suppose a uniform distribution of the real-time demand, with a probability of 25% for each demand scenario[21]. Parameters of the reseller's demand function are set by admitting that the maximum price he is willing to pay corresponds to the market price cap.

Table 1. Input parameters of the models

Parameter name	In the model	Value
Risk aversion coefficient for producers	A_c	0.0001
Risk aversion coefficient for reseller	A_r	0.0008
Fixed price for consumers (€/MWh)	P^v	120
Average total future demand (MWh)	$\overline{D_w}$	202.5
Parameters of cost function (€/MWh)	m_c	20, 40, 60 and 80[22]
Parameters of cost function (€/(MWh^2))	n_c	0.02
Parameter of supply function (€/(MWh^2))	b_c	0.02
Parameter of demand function (€/(MWh^2))	b_r	0.02

The two market configurations are defined as shown in Table 2. In the real-time market case, there is no penalty applied to imbalances. However, when a balancing mechanism is introduced, real-time market imbalances are penalized. We look at three cases regarding the level of the penalty. The low penalty is set at 10% level, the

---

[20]Since our analysis is a comparative one, model parameters are playing a secondary role as they are still constants when comparing the different designs.

[21]The four possible demand levels are: 180 MWh, 195 MWh, 210 MWh and 225 MWh.

[22]In the symmetric scenario, the parameter m_c for both producers takes the value 40. In the low-asymmetric scenario, it takes 40 for one producer and 60 for the other. Finally in the high-asymmetric scenario, it takes 20 for one producer and 80 for his competitor.

medium at 20% and the high at 30%. Finally, the three producers' cost structure scenarios are shown in Table 3.

Table 2. Definition of different real-time designs

Symbols	Configurations
k=0	Real-Time Market
k=0.1	Balancing Mechanism with low penalty
k=0.2	Balancing Mechanism with medium penalty
k=0.3	Balancing Mechanism with high penalty

Table 3. Definition of producers' cost structures scenarios

Symbols	Configurations
Sym	Symmetric producers
L-Asym	Low Asymmetric producers
H-Asym	High Asymmetric producers

Result 1: The implementation of balancing mechanism leads to a contracts increase in the forward market.

Figure 2 shows the optimal quantity exchanged between producers and the reseller in the forward market for each level of the penalty coefficient and regarding different scenarios of producers' cost structures. Firstly, we can clearly observe that commitments in the forward market increase with the level of the penalty. Indeed, given the fact that a balancing mechanism increases the volatility of imbalance prices in the real-time market, the reseller is pushed to buy more energy in the forward market in order to hedge against this risk. The reseller trades off between contracting energy at high prices in the forward market (as it is shown in the next sub-section) and reporting purchases in real-time and then facing prices volatility induced by the penalty. In a balancing mechanism, he has more incentives for securing his demand in the forward market.

Secondly, balancing mechanisms still ensure more exchanged quantity in the forward market regardless of whether producers are symmetric or asymmetric. We can notice that the more asymmetric the producers are, i.e. the higher the difference between producers' marginal costs is, the higher the forward quantity is, whatever the level of the penalty. Whether producers are symmetric or asymmetric and whatever their natures –base, shoulder or peak technology-, implementing a balancing mechanism would help to ensure more energy contracts in the forward market. For a thorough description of market players' incentives to rely more on forward contracts when a balancing mechanism is implemented, in the next sub-section we analyse forward energy prices under the different scenarios.
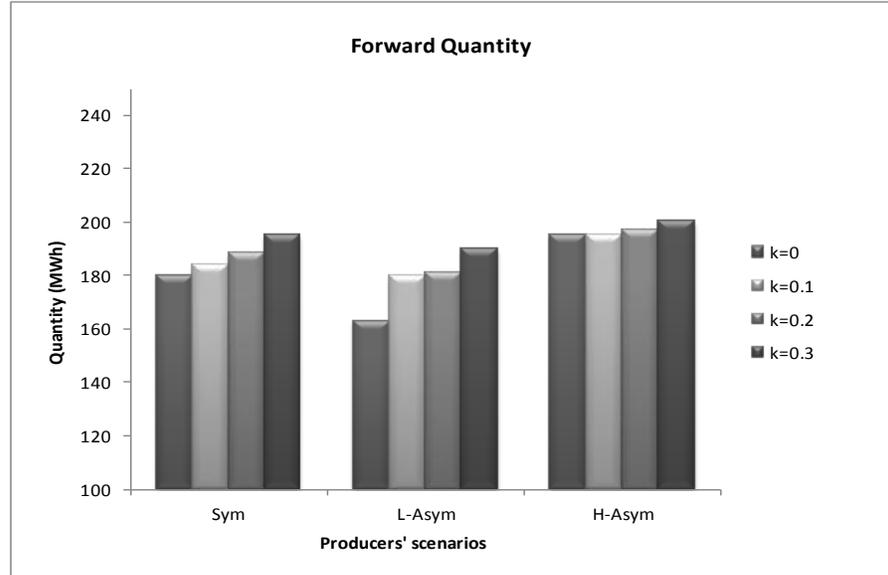
Figure 2. Forward contracts for each penalty scenarios and for each cost structure scenario

Result 2: Forward market prices are higher with balancing mechanisms and are principally set by producers' offers.

We can observe from Figure 3 that for the three scenarios of producers' cost structures, forward contracts are often exchanged at high prices and prices slowly increase when real-time imbalances are penalized. Penalizing the imbalances in real-time incites the reseller to buy more energy in the forward market. Accordingly, producers would require prices sufficiently high to accept scheduling more energy in the forward market. Producers would benefit from the reseller's needs of hedging in the forward market by offering high prices, even if the exchanged quantity rises. Consequently, forward price does not vary so much with the level of the penalty. The value of the penalty has only a small effect on the level of the forward price. If we link forward price evolution to forward quantity evolution (Figure 2), we can argue that while exchanged quantity in the forward market increases with the level of the penalty, prices are on the other hand distorted and mainly set by producers' supply functions. Whether a balancing mechanism or a real-time market is implemented, the reseller behaves as a quantity sensitive player, since he has no choice but to supply end consumers, while producers are price sensitive players since they globally face an inelastic demand.

Focusing now on forward prices differences as regard to the degree of producer asymmetry, Figure 3 also shows that forward prices decrease when the asymmetry increases. This result is expected given our assumption of oligopolistic Nash competition. With symmetric market players, market players who are price makers – like producers in the forward market of our case study- would have aligned interests to bid mutually for the highest price. When they are asymmetric, the less expensive producer would act more aggressively, benefiting from his cost advantage and bids for more quantity and a lower price compared to his competitor. This is how we record the lowest forward prices in the High asymmetric scenario. The ability of the

low cost producer –a base technology in this case- to accept selling at low price and offering so moderate individual supply function explains the drop in forward price compared to the low asymmetric and the symmetric cases.
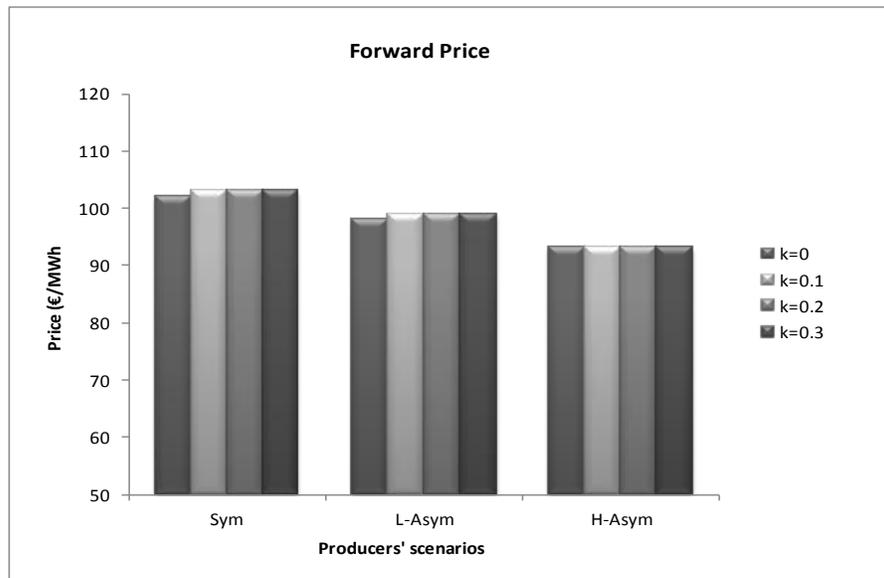

Figure 3. Forward energy prices for each penalty scenarios and for each cost structure scenario

Result 3: Average balancing prices decrease with penalties.

Figure 4 shows that in real-time, for all producers' cost structure scenarios, average balancing prices often drop when a balancing mechanism is implemented. In the symmetric case, the higher the penalty is, the lower the price is. When producers are asymmetric, prices weakly increase or still almost identical. This result is not surprising since the reseller is incentivized to secure his demand in the forward market when a balancing mechanism is implemented. Consequently, with a balancing mechanism the system in real-time is often in a situation of positive imbalance. Producers should consequently "buy" energy in order to balance the system which leads to lowering electricity prices in real-time.

These results confirm our theoretical predictions in section 3. When a real-time market is implemented, the imbalance is more likely to be negative than positive. From Lemma 1 and Lemma 2, whatever the degree of asymmetry, the balancing price corresponds to the upper limit of the producers' supply function[23]. Inversely, when a balancing mechanism is implemented, the system is often in a situation of positive imbalance, and as theoretically expected -Lemma 3 and Lemma 4-, producers offer to buy back energy at a minimum price constrained by the lower limit of their individual supply functions. We can finally observe that prices are the lowest in the high asymmetric scenario. Not surprisingly since in this scenario forward quantities are the highest.

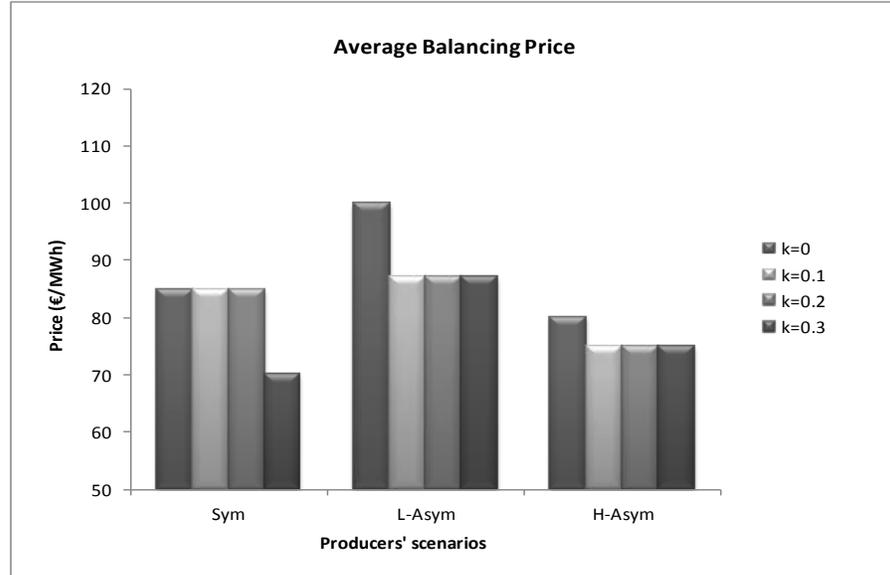[23]Producers offer the maximum value of $a_{c,RT}$.

Figure 4. Average spot market price for each penalty scenarios and for each cost structure scenario

Result 4: Implementing a balancing mechanism would increase market players' required forward premiums.

The forward premium is the difference between the forward and expected balancing prices - $P_F - E_w(P_{RT,w})$. It should compensate market participants for bearing the price or demand risk for the exchanges in the balancing market. It is mainly linked to the risk adverse behavior of market players and to the characteristics of the market design in place. Figure 5 points out that forward premium increases when going from real-time design to balancing mechanism design. Indeed, when a real-time market is implemented, the reseller has a little incentive to protect himself from balancing price risk. He reports his exchanges to the later market and fully experiences demand and price uncertainty. However, with a balancing mechanism, the forward premium increases in most of the asymmetry cases studied. A direct effect of penalties is to increase price volatility and so to incentivize the reseller to contract forward. Knowing that the reseller has no other choice, the producers exert their market power and the forward price is maintained high (as shown in the result 2's sub-section). Nevertheless, the balancing arrangement is then more often in a situation of positive imbalance with low real-time price, which then leads to raise the forward premium.
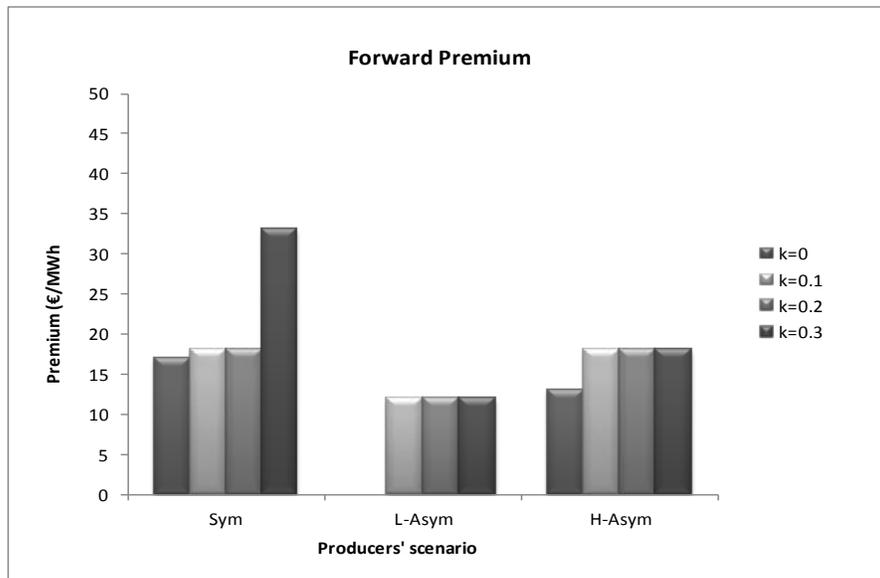
Figure 5. Forward Premium for each penalty scenarios and for each cost structure scenario

Result 5: Balancing mechanisms do not reduce the use of market power.

We use the Lenner Index to estimate producers' possible use of market power in both forward and balancing markets. From the figures below, we can observe first that market power is high, with or without balancing mechanisms and for both market steps. This is explained by the model assumptions that have led to prices almost close to upper price limits. However, as our study is comparative one, the aim is to look on the consequences of implementing balancing mechanisms, in terms of market power implications, instead of analyzing absolute values of the results. Having said that, we can observe that in the forward market, the Lenner Index does not depend on the penalty level. Implementing a balancing mechanism does not resolve the problem of market power in the forward market. One could say that producers make the most of resellers' need of covering themselves via the forward market by always manipulating the market prices. However, the index is weakly decreasing with the penalty in the real-time step. If we link this observation with the latter, we can argue that introducing a balancing mechanism would weakly shift the use of market power from the real-time market to the forward market.

Lenner index is also higher in the forward market compared to the real-time step. The gap is increasing when introducing balancing mechanisms. This is explained mainly by market players' behaviours when imbalances are penalized. On the one hand, producers react to the reseller's need to cover his demand in the forward step by manipulating the forward prices and requiring higher ones to accept contracting more energy. On the other hand, since with balancing mechanisms the system is more often in positive imbalance situations than not, electricity prices in real-time will fall to producers' marginal costs level which reduce the Lenner index at this step.
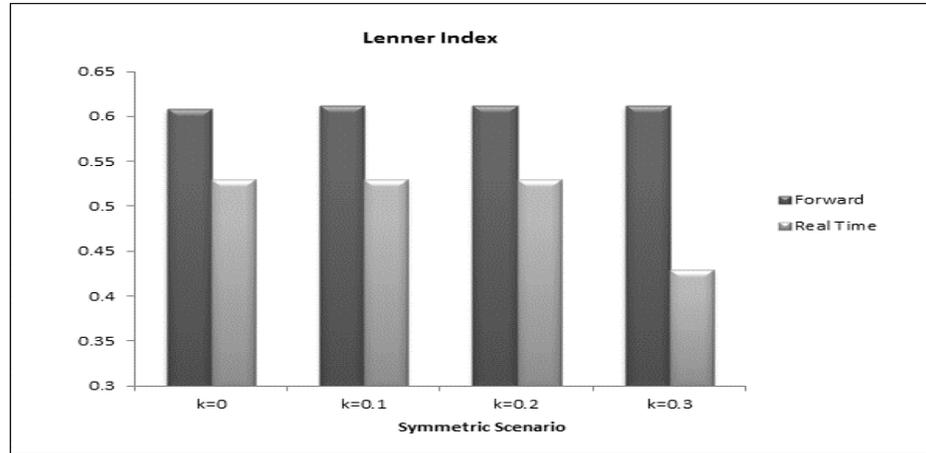
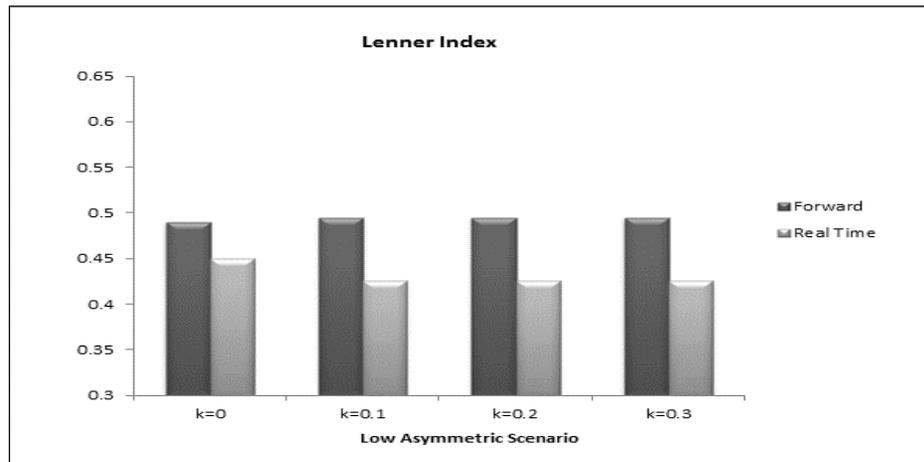**Figure 6. Lenner index in both markets with symmetric producers**

Figure 7. Lenner index in both markets with low asymmetric producers
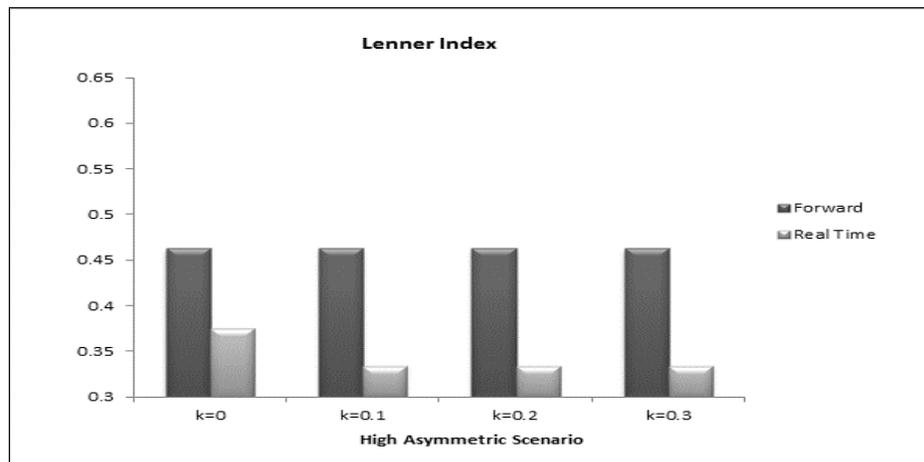

Figure 8. Lenner index in both markets with high asymmetric producers

To sum up: introducing a balancing mechanism should lead to increased energy exchanges in the forward market while neither electricity prices in both markets nor the possible use of market power could be strongly alleviated. The fact that producers are active in both markets while the reseller is active only in the forward market could significantly motivate this result. Price formations in the two markets are strongly linked with each other. Knowing that the reseller must satisfy his end consumers' demand whatever the price levels, producers act as price makers by setting the prices in both markets at high levels, benefiting from the reseller's need for quantity and also from their follower-behavior in the real-time market.

Result 6: Sensitivity analysis: varying the levels of the main parameters of the model would not modify the economic consequences of implementing a balancing mechanism. Only absolute values of the results could change.

In this section we study the sensitivity of our main results in regards to the changes of the levels of the parameters. We compare only two market design scenarios: the real-time market design and a balancing mechanism with a penalty equals to 15% with asymmetric producers.

We focus on four parameters. The first one is the risk aversion coefficient of the market players A_c and A_r. We vary these coefficients from a very low risk adverse behavior to very high risk adverse ones. The second sensitivity analysis focuses on the standard deviation of the real-time expected demand Std_D. An increase of this parameter makes demand more uncertain, which could be interpreted similarly as moving the temporal position of the gate closure[24] away from the balancing arrangement. We consider three scenarios of standard deviation for demand: 2% (low uncertainty), 10% (medium uncertainty) and 20% (high uncertainty).

The third and fourth parameters concern the slope of the individual supply and demand functions. We remind that the parameter m_c in the producer's cost function corresponds to the free variable a_c when determining its supply function. In the other hand, the slope of the marginal cost n_c and the slope of the supply function b_c are constant. Consequently, we study the sensitivity of our results to changes of the b_c parameters. Similarly, a sensitivity analysis is done on the slope of the demand function by varying the parameter d_c.

We can observe from figure 9 that when we raise the risk aversion coefficient of producers, quantities exchanged in the forward market increase. Market players' profits are uncertain due to the uncertainty of real-time demand. When they are risk averse, rational actors would prefer a smallest average but less volatile profit. In other words, they prefer a low but more certain profit than a high and volatile one. This translates into an increasing amount of forward contracts in our example. Quantities exchanged in the forward market are at least higher when a balancing mechanism is introduced. The difference is however less important when market players are more risk-adverse. They then undergo a high need to hedge their profit in the forward market, whatever the design of the balancing arrangement.

---

[24] The temporal position of the *gate closure* announces the end of the forward market. It results in final physical notifications which will be used to compute imbalances (see Glachant and Saguan (2005)).
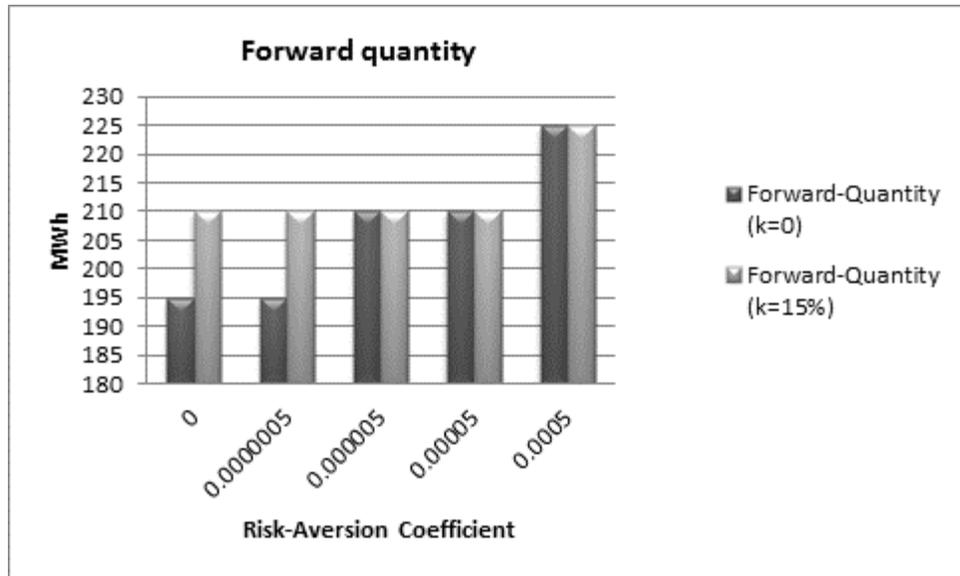
Figure 9. Sensibility of forward contracts to the average risk-averse coefficient

Not surprisingly, figure 10 shows that the forward premium increases when imbalances are more penalized. The degree of the risk aversion also affects the forward premium when it is particularly high.
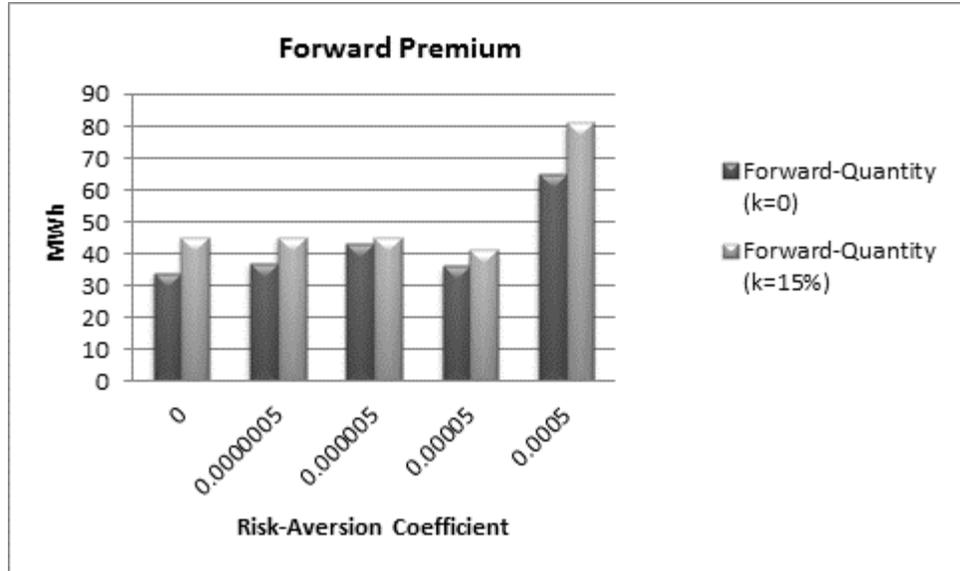

Figure 10. Sensibility of forward premium to the average risk-averse coefficient

Table 4 displays the effect of three levels of demand standard deviation on the variation of both forward quantity and forward premium when going from a real-time market design to a balancing mechanism design. We can observe that whatever the uncertainty on demand, both forward quantity and forward premium increase. The variation is however more important when the demand is more uncertain. This

_____

means that when the gate closure is placed farer from the real-time market[25],
resellers are more incentivized to hedge the risk due to uncertainty by contracting
more energy in the forward market. Producers would require a higher forward price
which leads to increase the forward premium level.

**Table 4. Sensitivity of forward quantity and premiums to demand standard
deviation**

Standard Deviation		2%	10%	20%
Going from a real-time market to Balancing Mechanism (k=15%)	Forward Quantity variation	2	3	10
	Forward Premium variation	3	20	25

Lastly, when we vary the slope of producers' supply function within a set of
possible levels of parameter b_c and go from a less-sensitive to a more-sensitive
offered quantity to prices, we observe that forward exchanges would globally
decrease when the sensitivity degree increases[26]. However, exchanged quantity with
a balancing mechanism is still higher than with a real-time market (Figure 11).
Forward premium is decreasing and falls to negative values for both designs of
balancing arrangement (Figure 12). Increasing this parameter graphically means that
the aggregate supply function is moved to the right hand side. This implies that new
market equilibriums in both markets are set at higher prices and lower quantity, all
other things being equal. This means also that any increase in the producers'
marginal cost function and consequently their supply function would reduce their
ability to offer in the forward market while they should require high market prices in
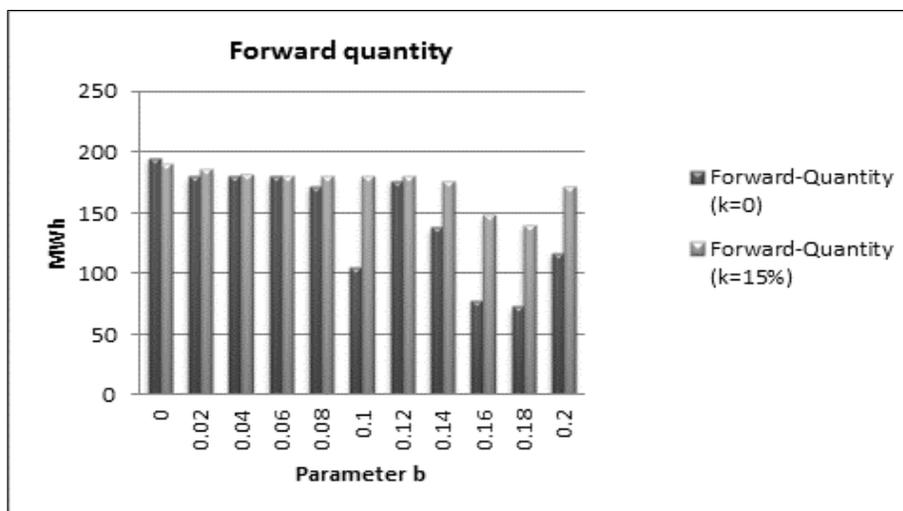both markets.

**Figure 11. Sensibility of forward contracts to the slope of the supply
Function / marginal cost function**

_____

[25]This is interpreted exactly by the increase of uncertainty on real time demand.
[26] Which also correspond to an increase in the slope of the marginal cost function of producers.
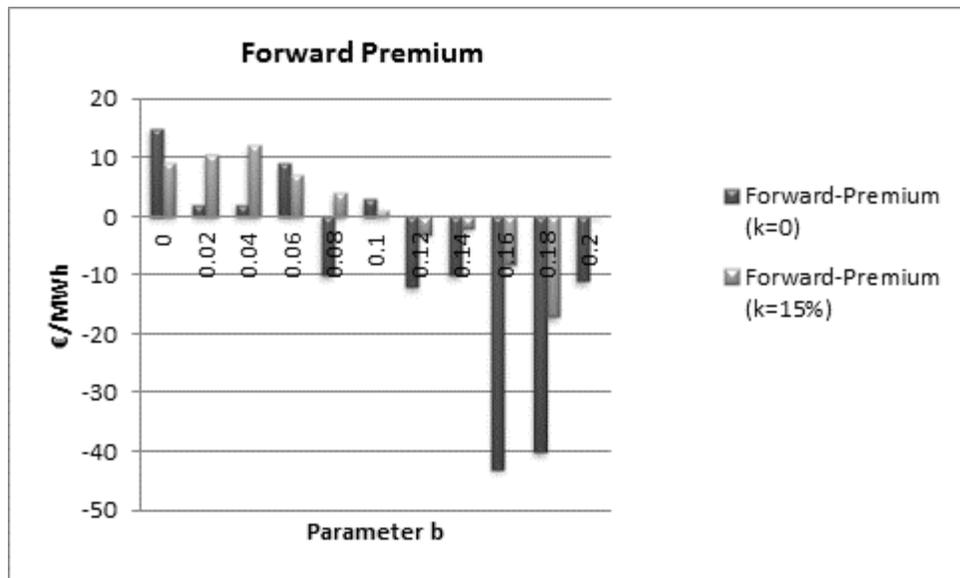
Figure 12. Sensibility of forward premium to the slope of the supply Function / marginal cost function

  The sensitivity of the results to the change of the slope of the demand function is however less significant (Figures 13 and 14). This confirms our observation in the Result 2 sub-section stating that the resellers are a quantity sensitive player while electricity prices in both markets are mainly set by the producers. Reseller's demand function does not affect significantly market equilibrium. Any shift in the demand function is accompanied by a variation of producers' offer to guarantee equilibrium stability around optimal levels for the producers. We can finally observe that whatever the scenario, forward quantity and forward premium increase when implementing a balancing mechanism.
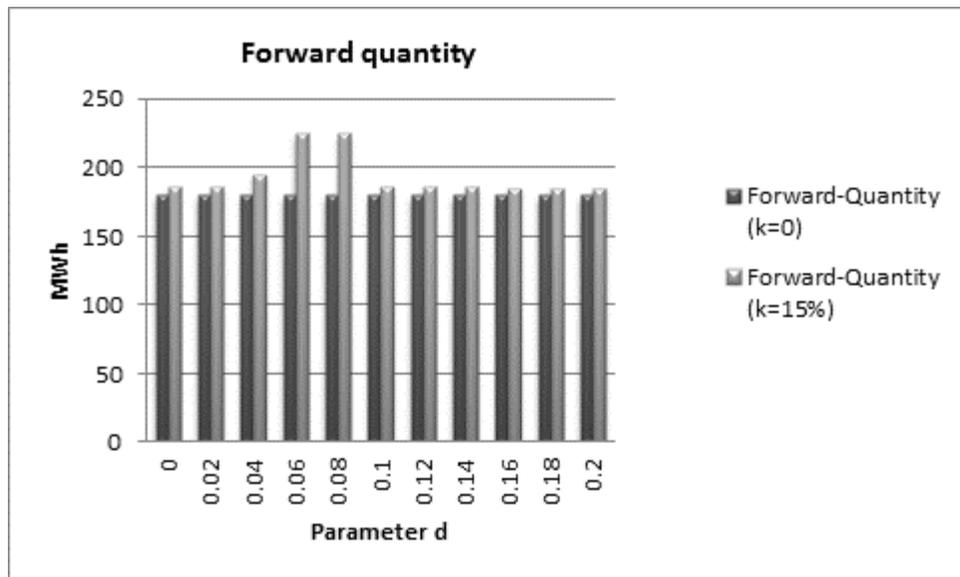

Figure 13. Sensibility of forward contracts to the slope of the demand function
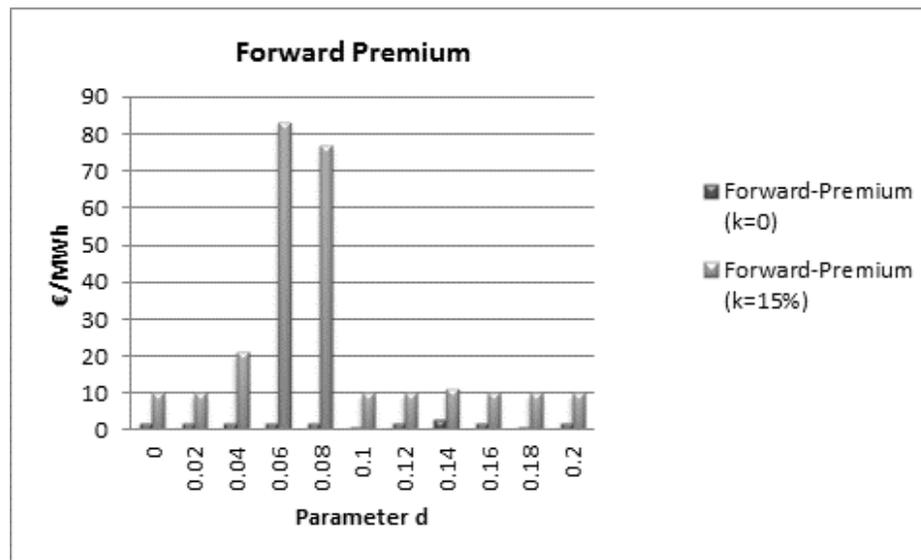
_____

Figure 14. Sensibility of forward premium to the slope of the demand function

To sum up: the sensitivity analysis done on the main parameters of the model show that only the absolute values of the results would change when varying the parameters while the economic effect of implementing a balancing mechanism on the forward and real-time equilibriums have not been significantly modified.

5. CONCLUSION

This paper considers the economic consequences of penalizing imbalances in the real-time market by introducing a kind of balancing mechanism. Two market designs have been compared. In the first design real-time exchanges are valued using the same market price, the called real-time market. In the second, real-time imbalances are however penalized, yielding different imbalance prices regarding the nature of the market player –producer or reseller- and his real-time position compared to his forward exchanges, the so-called balancing mechanism. We have implemented a two-stage dynamic model to consider the interaction between the forward market and the real-time market where market players –two producers and one reseller- compete in a Nash manner and relying on supply/demand function oligopoly competition. Based on some numerical examples, the main finding of the studies are: Firstly, implementing a balancing mechanism would increase forward contracts while raising electricity prices. Indeed, the reseller behaves as a quantity sensitive players and would be pushed to buy more energy in the forward market to secure his demand. Producers however are price-sensitive and would require always high prices to accept exchanging more energy in this market. Secondly, implementing a balancing mechanism does not reduce the use of market power. Producers make the most of resellers' need to cover themselves via the forward market by continuously manipulating the market prices.

This analysis could be extended in several ways. First, an introduction of an investment stage could be helpful to better understand market participants' behaviors in both markets when they are constrained by their capacity addition capabilities. Second, it would be relevant to increase the number of players –both producers and resellers- and suppose that resellers are also active in real-time market –by supposing that a part of demand is elastic. The case of making producers responsible for real-time imbalances should also be investigated if one analyzed how balancing mechanisms could help to improve generation forecasting in the short term.

REFERENCES

Allaz, B. "Oligopoly, Uncertainty and Strategic Forward Transactions", Internal Journal of Industrial Organization, 1992, 10: 297–308.

Anderson, E. and H. Xu. "Supply function equilibrium in electricity real-time markets with contracts and price caps", Journal of Optimization Theory and Applications, 2005, 124, 257–283.

Anderson, E., and H. Xu. "Finding supply function equilibria with asymmetric firms", 2006, Australian Graduate School of Management working paper, Sydney, Australia.

Bazaraa, M.S., Sherali, H.D., Shetty, C.M. "Nonlinear Programming: Theory and Algorithms", 1993, Second Edition, JohnWiley & Sons, NewYork.

Bertsekas, D.P. "Dynamic Programming and Optimal Control", 2000, 2nd Edition, Athena Scientific.

Bessembinder, H. and Lemmon, L. "Equilibrium Pricing and Optimal Hedging in Electricity Forward Market", Journal of Finance, 2002, 57, 1347-1382.

Chuang, A.S., Wu, F. and Varaiya, P. "A Game-Theoretic Model for Generation Expansion Planning: Problem Formulation and Numerical Comparisons", IEEE Power Engeneering Review, 2001, 21(10).

Day, C. J., Hobbs, B. F. and Pang, J. S. "Oligopolistic Competition in Power Networks: A Conjectured Supply Function Approach", IEEE Transactions on Power Systems, 2002, 17, 597–607.

DGEMP, "Coûts de référence de la production électrique", 2003, DGEMP Report.

DIGEC "Pour de nouvelles installations, les analyses traditionnelles concluent à la convergence des coûts des différentes filières mais avec des biais méthodologiques", 1997, DIEGEC Report.

Harker, P.T. "Generalized Nash Games and Quasi-Variational Inequalities", European Journal of Operation Research, 1991, 54: 81-98.

Kamat, R. and Oren, S. "Two Settlements Systems for Electricity Markets under Network Uncertainty and Market Power", Journal of Regulatory Economics, 2004, 25(1), 5-37.

Khalfallah, M.H. "A Game Theoretic Model for Generation Capacity Adequacy: Comparison between Investment Incentive Mechanisms in Electricity Markets", Energy Journal, 2011, 32(4), 117-157.

Murphy, F.H. and Smeers, Y. "Generation capacity expansion in imperfectly competitive restructured electricity markets", Operations Research, 2005, 53(4), 646-661.

Neuhoff, K. Hobbs, B.F. and Newbery, D. "Congestion Management in European Power Networks: Criteria to Assess the Available Options", 2011, DIW Berlin Discussion Paper No. 1161.

Pineau, P.O and Murto, P. "An oligopolistic investment model of the Finnish electricity market", Annals of Operations Research, 2003, 121:123–148.

Rivero, E., Barquin, J. andRouco, L. "European balancing markets", 2011, Energy Market (EEM), 8th International Conference on the European, 333 – 338.

Saguan, M. Glachant, J.M. "Analyzing Economic consequences of Central Western Europe Balancing Arrangements", Market modeling of the Central Western European Electricity Market, 15-16 September 2005 K.U.Leuven.

Saguan, M. "L'analyse économique des architectures de marchés électrique. Application au market design du temps réel", 2007, Phd Thesis, University of Paris 11.

Ventoza, M., Denis, M. and Redondo, C. "Expansion planning in electricity markets. Two different approaches", Proceedings of the 14th Power Systems Computation Conference (PSCC), 2002, Seville.

Von der Rehr, N.H.M. and Harbord, D. "Capacity investment and competition in decentralized electricity markets", 1997, Memorandum from Department of Economics, University of Oslo, No 27.

Walliser B. "Les fonctions des modèles économiques", 2005, Working paper ENPC-CERAS, EHESS.

Willems, B., Rumiantseva, I. and Weigt, H. "Cournot versus Supply Functions: What does the data tell us?" Energy Economics, 2009, 31(1), 38-47.

Appendix A:
Calculating producers' total aggregate supply function in the real-time market stage

As the individual supply function takes this form: $P_{c,RT,w} = a_{c,RT,w} + b_c \cdot q_{c,RT,w}$ we can deduce that the producer c's offered quantity is: $q_{c,RT,w} = \frac{P_{c,RT,w} - a_{c,RT,w}}{b_c}$.

When the market is cleared, electricity price is the same for all market participants. That's to say that when aggregating the total quantity offered by both producers in the real-time market, we can assume that: $P_{c,RT,w} = P_{RT,w} \; \forall c$.

Aggregating the two individual supply functions, we find at the end the total supply function expressed by: $P_{RT,w} = a_{RT,w} + b_{RT} \cdot q_{RT,w}$

Where,

$a_{RT,w} = \frac{a_{1,RT,w} \cdot b_2 + a_{2,RT,w} \cdot b_1}{b_1 + b_2}$ and $b_{RT} = \frac{b_1 \cdot b_2}{b_1 + b_2}$

We note that total supply function and demand function specific to the forward market are determined using the same method explained above.

Appendix B:
Use of the MCP method to find Nash equilibrium at the real-time market stage

In the real-time market, each producer chooses his supply function so that his own profit is maximized. Formally, the Nash market equilibrium defines a set of supply function levels such that no firm, taking his competitors' decisions as given, wishes to change his decision unilaterally.

In case of negative imbalance –section 3.2.1- each generator maximizes his profit (11) under constraints (12), (13) and (14). The decision variables are the quantity produced $q_{c,RT,w}$ and the price variable specific to the supply function of producer'c $a_{c,RT,w}$.

To state the model as an MCP problem we need to reformulate the optimization problem of each producer as follows:

B.1: Setting the first order optimality conditions associated to each producer's program: The Karush-Kuhn-Tucker's conditions

To calculate the optimality conditions of each program, we define first the Lagrangien function of the corresponding optimization problem :

$$L_{c,RT,w} = -P_{RT,w} \cdot q_{c,RT,w} + CT_{c,RT,w}\left(q^*_{c,F}, q_{c,RT,w}\right) - \beta \cdot \left(D_w - \sum q^*_{c,F} - \sum q_{c,RT,w}\right) - \alpha_c \cdot \left(a_{max} - a_{c,RT,w}\right) - \gamma_c \cdot \left(a_{c,RT,w} - a_{min}\right)$$

Then, we calculate the gardient of the Lagrangien function with respect to the two decision variable $q_{c,RT,w}$ and $a_{c,RT,w}$:

$$\frac{dL_{c,RT,w}}{dq_{c,RT,w}} = -\frac{dP_{RT,w}}{dq_{c,RT,w}} \cdot q_{c,RT,w} - P_{RT,w} + \frac{dCT_{c,RT,w}}{dq_{c,RT,w}} + \beta$$

$$\frac{dL_{c,RT,w}}{da_{c,RT,w}} = -\frac{dP_{RT,w}}{da_{c,RT,w}} \cdot q_{c,RT,w} + \frac{dCT_{c,RT,w}}{da_{c,RT,w}} + \alpha_c - \gamma_c$$

Optimality conditions of each producer's program are:

$$\frac{dL_{c,RT,w}}{dq_{c,RT,w}} \geq 0 \ ; \ q_{c,RT,w} \geq 0 \ and \ \frac{dL_{c,RT,w}}{dq_{c,RT,w}} \cdot q_{c,RT,w} = 0$$

$$\frac{dL_{c,RT,w}}{da_{c,RT,w}} = 0$$

$$\left(a_{max} - a_{c,RT,w}\right) \cdot \alpha_c = 0$$

$$\left(a_{c,RT,w} - a_{min}\right) \cdot \gamma_c = 0$$

$$a_{min} \leq a_{c,RT,w} \leq a_{max}$$

$$\sum q_{c,RT,w} = D_w - \sum q^*_{c,F}$$

$$\alpha_c \geq 0 \ ; \ \gamma_c \geq 0 \ and \ \beta \ free$$

This set of equations consists of the first order conditions multiplied by their corresponding decisions variables and the inequality constraints multiplied by their corresponding dual variables, all equal to zero; next the inequality constraints themselves and the equality constraint; and finally, the explicit statement of dual variables.

Grouping together all these conditions leads to an MCP problem. Equations (13)-(17) are therefore the solutions of this MCP problem. We note also that solutions for the positive imbalance scenarios are determined similarly to the negative imbalance scenario, by minimizing producers' payments instead of maximizing profit.

B.2: Existence and Uniqueness of the solution

Since that the cost functions are convex and continuously differentiable, the KKT conditions presented above are necessary and sufficient for optimality since the objective function is concave and the feasible region is polyhedral (Bazara et al. 1993).

We note that the constraint (9) is identical for all generators' programs. This leads to a generalized Nash equilibrium[27]. In this case, we make the assumption of an identical dual variable corresponding to this constraint and for all players in order to assure the uniqueness of the solution (Harker 1991).

---

[27]This is the case of a non cooperative game where players' strategies are not necessarily defined as an independent feasibility set.