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A Supply Function Equilibrium Model with Forward Contracts - An Application to Wholesale Electricity Markets

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ABSTRACT

Research into modeling electricity markets is continuing and the subject of many debates. All types of competition (Cournot, Bertrand, supply function) are utilized and have their advantages and disadvantages for electricity markets. It is well-recognized that models cannot address all questions of interest; however they appear as an interesting tool for gleaning insights into the complexity of electricity markets and whether electricity markets may deliver the expected benefits of liberalization. In particular, proving the existence of market power is a very complex task. Market simulation models should not be seen as the ultimate solution but as one powerful tool. While it is extremely difficult to prove if any market participants were manipulating markets, simulation models can show (under certain assumptions) if it would have been profitable to do so. Such models can be used in addition to traditional competition analysis. For instance, a model can estimate different benchmarks (competitive, supply function) against which actual market prices may be compared. This paper provides a practical application of the SFE concept and how such theoretical approach can be used in practice. The model combines the supply function equilibrium approach in an expanded version of the Baldick *et al.* model (2000) with forward contracting based on the model of Newbery (1998). We also discuss the different options for market modeling with respect to strategic variables and forward contracting. Finally, we present an application of this model to the Dutch electricity market.

Keywords: Supply function, equilibrium technique, market power, strategic bidding

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1. INTRODUCTION

The introduction of competition in the electricity industry is not an end in itself but represents the chosen tool by several countries to improve the overall efficiency of their electricity sector and is expected in turn to benefit consumers. These benefits are expected in terms of long run efficiency gain, technical innovation and efficient investment. However, the unusual characteristics of electricity supply and demand and the historically concentrated market structure of this industry have raised questions about the extent of competition. In particular, potential abuses of market power that can lead to prices above competitive levels and harm consumers represent a major concern for regulatory authorities and policy makers.

This issue has been illustrated by several recent international experiences. The analysis of market power has been the subject of an important literature in California, England & Wales, Scandinavia, and Spain.[1] In contrast to the US where FERC, and numerous other regulators have conducted investigations into the existence of dominant generators and important price spikes, market power has received little attention in Europe. In particular, large price spikes in the Netherlands, France, Germany and Denmark[2] have raised questions about the functioning of these markets and potential market manipulations. Surprisingly, despite several allegations of price manipulation and strategic bidding only Nord Pool started formal investigations.[3] In this paper, we show that, in those situations, market simulation models represent a useful tool for determining potential exercise of market power.

First, we discuss the different options for market modeling with respect to strategic variables and forward contracting. Second, we present a model combining the supply function equilibrium approach in an expanded version of the Baldick et al. model (2000 and 2004) with forward contracting based on the model of Newbery (1998). Finally, we present an application of this model to the Dutch electricity market.

2. MODELING STRATEGIC BEHAVIOUR IN ELECTRICITY MARKETS

2.1 Cournot and Bertrand competition

In order to represent and analyze the complexity of electricity markets, several simulation models have been developed. Simulation models appear as an interesting tool to test the hypothesis related to the possible behaviour of participants, to rank different design options, and to quantify the expected costs and benefits of any market structure change. They can also be used for forecasting purposes. It is worth

[1] For California, see Joskow and Kahn (2001), Harvey and Hogan (2000), Borenstein (2002) and Borenstein, Bushnell and Wolak (2002). For England and Wales, see Green and Newbery (1992), Wolak and Patrick (1997), and Wolfram (1998). For Scandinavia, see Andersson and Bergman (1995), Halseth (1998) and Hjalmarsson (2000). For Spain, see Rivier *et al* (1999) and Ciarreta and Espinosa (2003).

[2] During summer 2003, prices above 1800 Euros/MWh on the Dutch power Exchange, above 1000 Euros/MWh on the French power exchange, above 400 Euros/MWh in Denmark (part of Nord pool) and above 300 Euros/MWh on the German power exchange have been observed.

[3] See www.Nordpool.no.

noting that a single model cannot address all questions of interest, therefore for each model a clear purpose must be defined. Moreover, the models should not be viewed as substitutes for critical judgment by the analyst but as a complementary tool that helps the analyst to improve his judgment and intuition. Hence, in the case of models studying market power issues, careful modeling of electricity markets can provide valuable insights about the performance of markets, i.e., assessing whether electricity markets deliver the expected benefits of liberalization. In this section we discuss the different options for modeling behaviours of electricity markets participants.

Many electricity markets are characterized by a high level of concentration together with an inelastic demand, which makes them particularly sensitive to abuse of market power. Therefore, the theories of oligopoly competition provide suitable frameworks for analyzing these markets. Oligopoly competition covers the broad range of competition between the two extreme cases, perfect competition and pure monopoly. By contrast with perfect competition and pure monopoly, which are clearly defined, there exist a wide variety of theoretical frameworks for oligopoly competition.[4] Bertrand and Cournot competition represent the two extreme alternatives. Cournot models assume that each firm chooses a level of output taking rivals' production decisions as given (Cournot, 1838). In such a model, generators compete on quantity. By contrast, Bertrand models appeal to a contrasting notion of rivalry, using prices rather than quantity (Bertrand, 1883). Under Cournot competition, prices usually exceed short-run marginal costs due to important incentives to withhold capacity for generators. Under Bertrand competition, market outcomes are close to the results of the perfectly competitive market, i.e., the electricity prices equal the short-run marginal costs of generating power (Tirole, 1988).[5]

In theory, depending on the purpose of the model and the type of market, one modeling approach might be more appropriate than another. In the case of electricity markets, on the one hand, it has been argued that since electricity is a non-storable good, i.e., production has to be sold instantaneously, Bertrand-type competition appears as a suitable assumption for generators. On the other hand, and especially in period of high demand,[6] the Cournot paradigm corresponds more closely to electricity markets. In the Bertrand approach, any firm can capture the entire market by pricing below other competitors but, since electricity producers have increasing marginal costs and limited installed capacity, the simple Bertrand assumptions regarding behaviour appear less realistic than those of Cournot. Therefore an important literature based on various forms of Cournot-type strategic behaviour in electricity markets is available.

For instance, Andersson and Bergman (1995) studied the relation between equilibrium price and the number and size distribution of firms on the market using Cournot assumptions. Borenstein *et al.* (1997) used a Cournot model to analyze the potential for market power in New Jersey (US). Oren (1997) used the Cournot

[4] For instance, the number of generators can vary (two, three, four…ten), the respective sizes of generators (symmetric or asymmetric), the type of competition (Cournot, Bertrand, Stackelberg…), market restrictions (limited capacity, barrier to entry,…).

[5] This is strictly true only under restrictive assumptions such as for instance no capacity constraints and constant marginal costs.

[6] Which are potentially more vulnerable to abuse of market power.

framework to analyze the impact of a new transmission rights scheme and show how generators may obtain the congestion rent of a transmission line. Similarly, Hogan (1997) developed a Cournot model in the presence of transmission constraints to analyze the behaviour of dominant firms. Borenstein and Bushnell (1999) used historical cost data to simulate the California electricity market assuming static Cournot competition. Smeers and Wei (1999) and Hobbs *et al.* (2002, 2003) assume Cournot competition[7] to model an oligopolistic European electricity market and analyze potential inefficiency of transmission pricing. Younes and Ilic (1997), Berry *et al.* (1999), Stoft (1999), Cunningham *et al.* (2002) and Willems (2002) also used this approach to analyze the relationships between market power and transmission constraints, e.g., how congestion potentially creates submarkets that could encourage strategic behaviour. Nevertheless, in some circumstances, e.g., (periods of low demand), Hobbs (1986), Aghion and Bolton (1987), and Wolfram (1998) have suggested that Bertrand models might be a relevant approach.

While Cournot models are relatively flexible and tractable, they are not realistic methods for modeling strategic interactions competition in most European markets.[8] For instance, Harvey and Hogan (2000) argued that the Cournot formulation was usually justified based on its analytical convenience rather than its descriptive power. Cournot models are based on pure quantity bids, but in most electricity markets strategies of firms are actual non-decreasing functions from price to quantity.[9] In order to capture this common feature of electricity market architecture, the concept of "supply function equilibrium" (SFE) has been presented as an alternative approach for modeling strategic interaction. Especially for the analysis of market power in electricity markets, Kahn (1998) argued that, while the flexibility of the Cournot approach made it attractive to the analyst, the SFE approach was conceptually superior.

2.2 The case for supply function equilibrium competition

The SFE approach advanced by Klemperer and Meyer (1989) yields market outcomes that lie between those of the Bertrand and Cournot approaches. The SFE approach assumes that facing uncertain demand, rather than competing only with fixed prices or quantities, generators will compete in price and quantity schedules, i.e., supply functions. The idea is that, when firms must choose their strategies before knowing what the realization of uncertain demand will be, they will define an entire supply curve with different prices for different quantities. Klemperer and Meyer showed that for a given demand function for any price above the competitive one, there exists a corresponding Nash equilibrium. The interesting feature of the SFE approach is its capacity to accommodate random shocks in demand. Moreover, the SFE approach appears more realistic than the "single variable" approaches because the SFE assumption closely reflects the bidding rules observed in most

[7] Hobbs *et al* (2003) also used the supply function approach.

[8] Hobbs (2001) also recognized the lack of realism of Cournot assumption for the western US market.

[9] All European power exchanges use price-quantities bids (e.g., Nord pool, The Netherlands, United Kingdom, France, Germany).

organized markets.[10] This allows a better understanding of companies' bidding behaviours, in particular in markets where they bid repeatedly.

The SFE approach has therefore been used for a number of important analyses. Green and Newbery (1992) and Bolle (1992) were the first to employ the supply function equilibrium approach in electricity markets. Their objective was to estimate the level of competition in the British electricity spot market. Bohn *et al.* (1999) adapted the supply function approach for the analysis of the California Power Exchange. In particular, though they did not analyze a dynamic game, Bohn *et al.* took into account the fact that in California, a firm can bid a different curve for each period while in the former British pool firms needed to make a single bid for all 48 defined periods of the day. Rudkevich (1999) have also used the supply function approach to analyze strategic bidding and to attempt to predict joint behaviour of market participants. Day *et al.* (2001) have extended the supply function approach by introducing the anticipation of firms concerning the output of rivals (Conjectured Supply Function). Hobbs and Rijkers (2002) have applied a similar model to the Benelux, French and German markets in order to analyze the (in)efficiency of transmission pricing in Europe. In Europe, some analyses have also been done recently in Spain. Ciarreta and Espinosa (2003), for example have analyzed the performance of the Spanish pool using a supply function equilibrium approach. To measure market power Ciarreta and Espinosa compare the behaviour of firms under the ownership of larger generators and the others under the ownership of smaller firms. They concluded that large generators were exploiting their market power and were consistently submitting supply curves with higher prices than their competitive benchmark.

The SFE models have been traditionally considered as having limited applicability because of their computational intractability. In general all supply function equilibria are bounded by the Cournot and Bertrand equilibria. While Klemperer and Meyer have demonstrated conditions for the equilibrium to be unique, these conditions are restrictive. Therefore the main problem with SFE models is that in general all equilibria between these two extremes are possible.[11] However, in practice a single equilibrium can be selected by imposing more restrictive assumptions. For instance, Green and Newbery (1992) chose to focus on the least competitive SFE for the British electricity market.[12] Green (1996) assumed a linear supply function.[13] Berry *et al.* (1999), Rudkevich (1999), and Baldick *et al.* (2000 and 2004) used affine[14] supply functions, to restrict the set of equilibria. Finally, Baldick and Hogan (2001) showed theoretically that even when there is a wide range of equilibria, all but one of these equilibria is unstable (with respect to a particular class of perturbations) which allows the identification of a single stable equilibrium.

The supply function equilibrium approach is therefore attractive compared to the Cournot approach for three main reasons. First, it offers a more realistic way of

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[10] See note 9 *supra*.

[11] In SFE equilibrium, price can range from perfect competition outcomes to the Cournot equilibrium price (Bolle, 1992).

[12] Green and Newbery (1992) also noted that the range of equilibria was limited in the presence of capacity constraints.

[13] That is, the intercept of the supply function is zero.

[14] That is, constant slope and an (possibly zero) intercept.

modelling how firms compete in the current electricity markets where suppliers submit non-decreasing functions in the price-quantity plane. Second, in the SFE approach, random shocks in demand can be easily accommodated which is especially important for electricity markets due to real-world demand uncertainty. This explains why SFE predictions are often closer to (though still typically higher than) observed prices while Cournot models generally produce worse predictions.[15] Finally, the SFE approach recognizes the fact that generators price their output prior to actually producing it, consistent with actual electricity market bidding protocols.

2.3 Modeling forward contracts

In most electricity markets, firms do not sell all their output to the spot market but use long-term forward contracts. Such a characteristic has an important impact on the competition and needs to be considered in any modeling attempt. The original rationale for the existence of forward markets is related to the hedging of risk, i.e., firms and consumers may want to smooth their cash flows and limit the impact of the short-term demand volatility.[16] In Europe, these types of contracts represent the largest share of total traded volumes. A second aspect is related to the incentives of the firms. Forward contracts reduce the incentives of the generator to push up the prices by withholding output that clears the spot market because the generator does not receive the higher spot price on the output it has already sold through forward contracts.[17] Therefore, an analysis that includes forward contracts concludes that in general (at least in a static game), forward markets tend to reduce market power.[18] Allaz and Vila (1993) showed that the existence of forward contract changes strategic incentives of the generators in a way that enhances competition and efficiency.[19]

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[15] See Frame and Joskow (1998).

[16] See Newbery and Stiglitz (1981), Anderson and Danthine (1983), Newbery (1984).

[17] However, one can also argue that if the forward contracts use the spot prices as an index, generators may still have incentives to exercise market power in the spot market and benefit from it in future forward trades, e.g., if the forward contracts of year N+1 are indexed on the spot price in year N, exercising market power in the year N will have no influence on previously signed forward contracts but can be beneficial for future forward contracts (i.e., N+1, N+2…). Hence in practice, generators do not only choose their strategies in a period N only considering the effect it would have on this period price and profitability, but also considering the indirect effects on the next period price and profitability. See Powell (1993) and Green (1996) for a discussion.

[18] See Harvey and Hogan (2000) for a critical analysis of this conclusion.

[19] It is worth noting that the assumptions of this model are particularly restrictive. In particular, Allaz and Villa (1993) assume (in their main model) a one-period game with two generators in which each generator takes the output of its competitor as fixed. In this context the introduction of forwards contracts leads to a prisoner's dilemma, i.e., each generator has the incentive to trade forward but when they both do so, they end up worse. This beneficial effect on competition depends critically on the assumption that the game is played only once. In a dynamic game, a prisoner's dilemma can produce very different outcomes (e.g., collusion).

Borenstein (2002) argued that long-term contracting in the Californian market[20] would have helped to mitigate market power. However, it is worth noting that most models considering forward contracts used the Cournot framework (Allaz and Vila, 1993; Bolle, 1993; Powell, 1993; Batstone, 2000) while supply function equilibrium models in general do not consider the impact of forward contracts (Rudkevich, 1999, Baldick *et al.*, 2000 and 2004; Bolle, 2001). Only the analyses of Newbery (1998) and Green (1999) combined forward markets with supply function competition in the spot market. These two papers consider the case of a duopoly where two types of markets coexist: a spot market and a forward market. In these models, competition in the forward market is modeled as Cournot competition, i.e., in the forward market each firm offers a fixed quantity of contracts. While it appears as natural to use the supply function approach for both markets, in practice the interaction between the two markets would lead to a double infinity of solution, as Newbery (1998) observed.[21] Therefore, Cournot competition is usually used in the forward markets. From a modeling point of view the interaction between forward and spot markets represent a major challenge.

Newbery (1998) follows the assumption of Allaz and Villa (1993) arguing that the existence of forward contracts reduce the incentives of generators to act strategically on the spot market since they would only benefit from high prices for the residual part of output which has not been sold in the forward market. In the case where a generator has fully contracted all its output in the forward market, its best strategy (under certainty) is to offer into the spot market at marginal costs, i.e., the profitability of the generator would have been determined in the forward market and the spot price will have no influence on it.[22] However, if a generator is fully contracted, a more aggressive behaviour in the spot market can be expected which in turn will encourage competitor to sell further forward contracts. In the symmetrical duopoly case, an interesting conclusion of Newbery is that if each generator assumes that their level of forward contracts offered has no impact on the level offered by their competitors, no generator will offer forward contracts.

Green (1999) used supply functions to model the British spot market and modeled the forward market with conjectural variations. In a first stage, firms simultaneously choose quantities to be sold in the forward markets. In a second stage, firms submit simultaneously supply functions in the spot market.[23] Similar to Newbery (1998), Green found that a firm will bid at marginal cost on the spot market for the quantity equal to the amount covered by its forward contracts and that a rational decision for a firm with Cournot conjecture on the forward market

[20] In California, generators were required to submit all generation bids into the wholesale spot market (i.e., the power exchange) and thus face the volatility of fluctuating prices—forward contracting was prohibited.

[21] For each spot market equilibrium there exists a continuum of equilibria in the forward market, and for each equilibrium in the forward market there exists a continuum of equilibria in the spot market.

[22] See note 19 *supra.*

[23] From a modeling point of view, the debate about the realism of the sequence first "selling forward" then "selling spot" against first "spot" then "forward" is in some way similar to the "chicken and egg" story. In this paper we follow the traditional sequence where firms first define the quantities they are willing to sell in the forward market.

will be to not sell any forward contracts unless this will affect its competitor's spot market strategy.[24]

3. A SUPPLY FUNCTION EQUILIBRIUM MODEL

3.1 Introduction

In this section we present a general description of the "Simulation of bidding actions and decision" model (SYMBAD[25]). This model is based on the supply function equilibrium approach for simulating the behaviour of the participants in an electricity market. The framework of this model is based on the approach of Baldick *et al.* (2000 and 2004)[26] and Rudkevich (1999). This approach is especially interesting because it provides the explicit expression for computation of the Nash equilibrium for an asymmetric oligopoly.[27] First, the model allows the use of piecewise affine supply functions which permits a more precise approximation of marginal costs, in case the affine approximation is not appropriate. Second, several demand segments, including different demand slopes, can be defined. Third, generation capacity constraints are considered. Finally, we attempt to include forward contracts in the model and assess their influence on market power.

The first important issue in electricity market modeling is the definition of generator's marginal cost curves. In practice, market suppliers submit a non-decreasing set of bid schedules containing price and quantity. In order to define a unique equilibrium, affine cost curves are characterized mathematically through regression analysis and linear approximation.[28] In the case where linear regression does not present a sufficient level of accuracy, piecewise affine approximation is used.[29] The great advantage of using linear (or piecewise linear) marginal cost curves are their ability to model asymmetric generators which is an important characteristic of most real electricity systems. For each generator, then a marginal cost curve is therefore defined as described above. In the following step, the individual marginal costs curves are summed up to derive the aggregate marginal cost curve. This aggregate curve can be considered as a "competitive benchmark" for the spot

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[24] If the rival responds to an increase in the contract sales of the firm by selling fewer contracts, then this will make the rival less aggressive in the spot market, allowing the first firm a greater market share. If the contract sales of the rival do not respond to that of the first firm, then selling contracts can still be optimal if this will affect the spot market strategy of the rival (Green, 1999).

[25] SYMBAD was developed by, and is a proprietary model of KEMA Consulting.

[26] The method used to find an approximation to an SFE is very similar to that of Baldick et al. (2000 and 2004) and is essentially ad-hoc to that method. An alternative method (Baldick and Hogan, 2002) is to use an iterative computational approach, though this has the disadvantage of being computationally intensive. A more recent work by Holmberg (2005) proposes a numerical approach using systems of ordinary differential equations (ODEs) to solve the SFE problem.

[27] While Green and Newbery (1992) considered a symmetric duopoly with linear supply and demand functions and obtained the explicit expressions for computation of the Nash equilibrium, Green (1996) was the first who examined the asymmetric case, and Baldick *et al* (2000) generalized their results for asymmetric oligopoly.

[28] We derive from the horizontal segments generator costs a set of points. These points are then used for a single linear regression. The description and nature of the engineering cost used will be described in section 4-2.

[29] In this case, a polynomial approximation is used which then subsequently is approximated with piecewise linear functions.

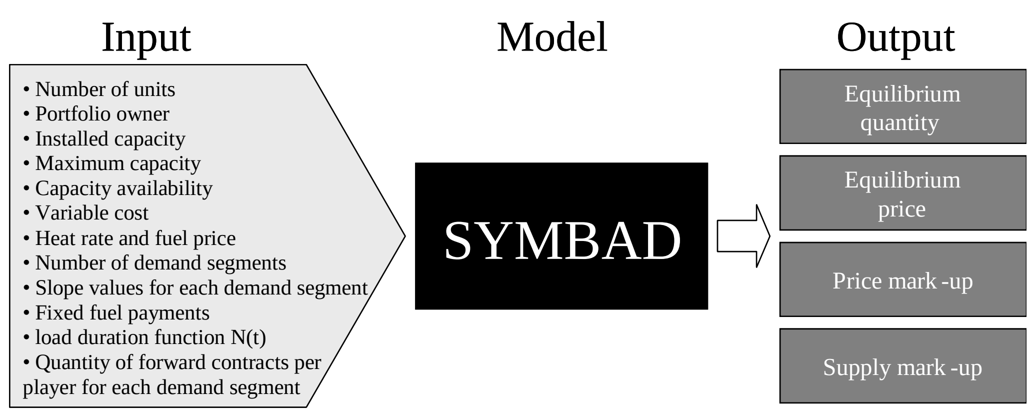
market, i.e., representing marginal cost bidding in aggregate. Next, the supply function curves are defined for each market participant in accordance with the linear SFE approach. This assumes that each participant bids in Nash equilibrium according to its best SFE strategy.

Different demand segments are defined to recognize the important changes in demand between different periods. These demand segments are mainly defined by the slope of the demand curve (elasticity). The intersection between the demand curves and the aggregate marginal cost curve and between the demand curves and the aggregate supply function define the equilibrium for the "competitive scenario" and the "strategic bidding scenario" respectively. For each demand segment the difference between the outcomes of the two scenarios defines the price mark-up. This price mark-up can be interpreted as a measure of potential market power.

In the model, forward contracts are considered to take into account the fact that, in practice, generators sell a large fraction of their output in advance. For this purpose we assume that each generator sells in the forward market an exogenous quantity which is publicly known before bidding into the spot market. The model extends the approach of Newbery (1998) for *n*-non-symmetrical firms. This approach is based on maximization of a generator's profit function that considers gains and losses of generators due to the difference between the spot and forward prices. In the model, these modified profit functions are defined for each generator. To summarize, SYMBAD simulates the optimal bidding behaviour of specified generators, optimizes the expected mark-ups of bid prices in the market and predicts market prices under different bidding strategies.

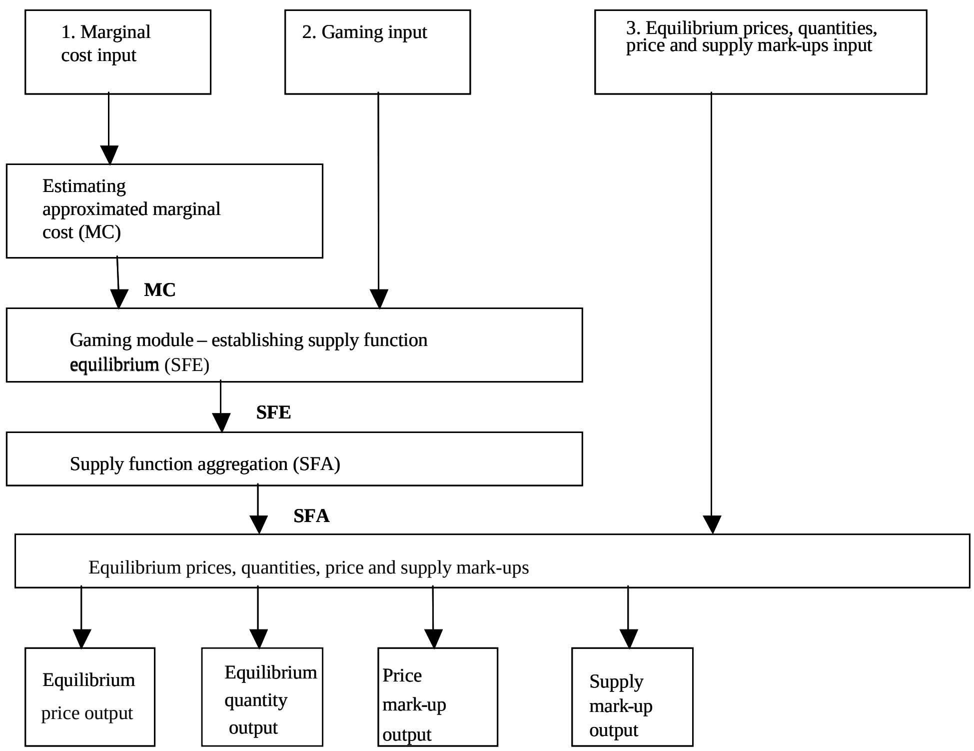
The SYMBAD model simplifies the supply function approach to electricity markets by restriction to piecewise linear supply functions. Such a simplification suffers principally from the fact that the players in real electricity markets have not such a restriction. In addition, in the model specification, the piecewise linear function of one player is not even a best response to the functions used by other players. This "double deviation" from theory may be justified under the aspect of simulating real markets if it is successful in producing properties (say price distributions) of real electricity markets.

Figure 1:
Simulation overview



Based on the input data shown in Figure 1 and three-segment piecewise affine approximations of generators' marginal cost functions, SYMBAD incorporates a new gaming procedure which computes Nash equilibria in an electricity pool with forward contracts. An overview of the SYMBAD algorithm dataflow is shown in Figure 2. The SYMBAD model is implemented in Mathematica.

Figure 2:
SYMBAD algorithm dataflow



In the following sections we describe the main aspects of SYMBAD's mathematical formulation.

3.2 General description

In SYMBAD we define the following variables and parameters:

p price,

t time,

$N(t)$ – load duration characteristic function, represents peak demand in each hour. That is, a set of values giving the maximum volumes of electricity that the consumers want to consume in each hour.

γ – demand curve slope,[30]

ε – exogenous shock to industry demand, where ε is a scalar variable with strictly positive density in the interval $[0; \infty)$,[31]

$D=D(p,\varepsilon)$ – industry demand, where for all (p, ε), $-\infty < D_p < 0$, $D_{pp} \leq 0$, $D_\varepsilon > 0$, and $D_{p\varepsilon} = 0$,[32]

$D(p,t) = N(t) - \gamma p$,

D_r – residual demand resulting from a price-taking fringe with capacity constraints,

$x_j \geq 0$ – the amount of forward contracts sold by generator j,[33]

$S_j(p)$ – the supply function (from price to quantity) for generator j, the supply function is continuous, twice differentiable, and monotonically increasing,

$S_{-j}(p) = \sum_{i \neq j} S_i(p)$ - the sum of supply functions excluding generator j,

$C_j(q)$ – the total cost function of generator j as function of produced quantity q is strictly increasing and strictly convex (i.e., $C_j'(q) > 0$ and $C_j''(q) > 0$ for all $q_j > 0$),

$C_j'(S_j)$ – marginal cost of generator j as function of j's production,

Firms choose supply functions simultaneously without knowledge of the realization of ε. After the realization of ε, the supply functions implemented by each firm produce at a point ($p^*(\varepsilon)$, $S_j(p^*(\varepsilon))$), such that:[34]

$$\sum_{j=1}^{n} S_j(p^*(\varepsilon)) = D(p^*(\varepsilon), \varepsilon) \qquad (1)$$

We assume that the shock in the model will vary over time (i.e., $\varepsilon \equiv t$). Because of this assumption, the conditions of the model of Klemperer and Meyer are satisfied. The equilibrium price $p^*(\varepsilon)$ is then a function of time t. So the solution of the condition:

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[30] We use constant demand slope in the SFE for analytical convenience. Otherwise the equations do not work out as nicely.
[31] This assumption is unrealistic in the context of electricity markets, but it is assumed for convenience and is from Klemperer and Meyer (1989).
[32] i.e., $D = f(p) + g(\varepsilon)$.
[33] To use the SFE model without forward contracts set x_j=0.
[34] There are two possible definitions of D. In the first definition $D=D(p,\varepsilon)=f(p)+g(\varepsilon)$ plus conditions for differentiation (Klemperer and Meyer, 1989). Here the exogenous shock is not precisely defined but rather it is a random variable that has an impact on the market. In the second definition $D=D(p,t)=N(t)-\gamma p$. It assumes that the time is the variable for the exogenous shock and that the demand function is affected by the load magnitude. This is the usual convention applied in the electric power literature. In this paper we have made them equivalent by setting $D(p,t=\varepsilon)=N(\varepsilon)-\gamma p$ such that $g(\varepsilon)=N(t=\varepsilon)$ and $f(p)=-\gamma p$.

$$\frac{dS_{-j}}{dp} = \frac{S_j - x_j}{p - C_j'(S_j)} + \frac{dD}{dp}, j = 1..n \textit{ for all } p > 0 \qquad (2)$$

will represent the supply function equilibrium for every hour in the period of bidding. Klemperer and Meyer's necessary conditions for supply function equilibrium under uncertainty are used in the assumptions about the demand in the SYMBAD model in the following way:

$$\frac{dS_{-j}}{dp} = \frac{S_j - x_j}{p - C_j'(S_j)} - \gamma, j = 1..n \textit{ for all } p > 0 \qquad (3)$$

Here demand is piecewise affine.

3.3 Supply function equilibrium in case of affine supply functions

Let there be *n* generators in the spot market and assume that marginal cost of each generator $C_j'(q)$ is affine i.e.,:

$$C_j'(q_j) = c_j q_j + a_j; j = 1..n, \qquad (4)$$

$c_j > 0$,

q_j – generator *j*'s quantity produced,

a_j – marginal cost intercept for generator *j*

c_j– marginal cost slope for generator *j*

Likewise, we assume that the supply functions $S_j(p)$ will be affine, i.e.,:

$$S_j(p) = \beta_j(p - \alpha_j), j = 1..n, \; p \geq \alpha_j, \; \beta_j > 0 \qquad (5)$$

$$S_{-j}(p) = \sum_{i \neq j} S_i(p) = \sum_{i \neq j} \beta_i(p - \alpha_i)$$

α_j – price axis intercept of the supply function of generator *j*

β_j – slope of the supply function of generator *j*

After substituting in Equation (3) we obtain the following result:

$$\sum_{i \neq j} \beta_i = \frac{\beta_j(p - \alpha_i) - x_j}{p - (c_j(\beta_j(p - \alpha_i)) + a_j)} - \gamma \textit{ ,j=1..n} \qquad (6)$$

which is:

$$\left(p-(c_j(\beta_j(p-\alpha_i))+a_j)\right)\left[\sum_{i\neq j}\beta_i+\gamma\right]=\beta_j(p-\alpha_i)-x_j\ ,j=1..n \qquad (7)$$

This equation must be satisfied for each value of p, where all of the generators take part in the game (i.e., $p \geq Max\{a_i\}$). This implies that the coefficient of price and the constant term in both sides of Equation (7) are equal.

$$\left(1-c_j\beta_j\right)\left[\sum_{i\neq j}\beta_i+\gamma\right]=\beta_j\ ,j=1..n \qquad (8)$$

$$(c_j\beta_j\alpha_j-a_j)\left[\sum_{i\neq j}\beta_i+\gamma\right]=-\beta_j\alpha_j-x_j\ ,j=1..n \qquad (9)$$

After solving the first equation for β_j and substituting β_j into the second equation we obtain that

$$\alpha_j=a_j-\frac{x_j}{\sum_{i\neq j}\beta_i+\gamma}\ ,j=1..n.$$

Now we must find the beta coefficient. The following lemma shows that the solution of the system Equation (8) is unique and gives an algorithm for obtaining these coefficients.

<u>**Lemma:**</u> If n>1 in system Equation (8) has a unique positive solution $\beta_1,..,\beta_n$ such that

$$\beta_j=\frac{1}{c_M}\left[s_j+\left(\frac{U+\gamma c_M}{2}\right)-\sqrt{s_j^2+\frac{(U+\gamma c_M)^2}{4}}\right],j=1..n \qquad (10)$$

where

$$\frac{1}{c_M}=\sum\frac{1}{c_i}$$

$$s_j=\frac{c_j}{\sum c_i}$$

$U = c_M \sum \beta_i$

U can be found from the following equation:

$U = 1 + n\dfrac{U + \gamma c_M}{2} - \sum \sqrt{s_i^2 + \dfrac{(U + \gamma c_M)^2}{4}}$, which has a unique positive solution for $0<U<1$ (Rudkevich, 1999).

3.4 The model with maximum capacity constraints

This version of the model includes an upper capacity constraint.

Let maximum capacity for every generator be $\overline{q}_j$, *j=1..n.*

So the question in this section is "What happens with the supply functions of all generators for values of p higher than the value for which exactly one of the generators reaches its maximum quantity?"

Let the first of the generators reach its maximum capacity at $p = p^*$. So its function will represent the piecewise affine left from p^*, and constant q_1 for $p > p^*$.[35]

What happens with the supply functions of the other generators?

The residual demand for $p > p^*$, becomes $D(p,t)$ - q_1 and the system of equations of the equilibrium slopes of the other generators will be:

$$\frac{dS_{-j}}{dp} = \frac{S_j - x_j}{p - C_j'(S_j)} - \frac{d(D(p,t) - q_1)}{dp}, \quad j=2..n. \tag{11}$$

which is the same as:

$$\frac{dS_{-j}}{dp} = \frac{S_j - x_j}{p - C_j'(S_j)} - D_p, \quad j=2..n, \tag{12}$$

so we solve the same system of equations but with one less equation and one less parameter.

It can be demonstrated that as the number of the variables and equations in this system increases, their slopes increase too. So for every of the generators in the right interval the slope will decrease. This means that for each of them the left intercept will be greater from the right intercept. In the previous section we managed with a similar intercept as we assumed that for price p^* we can sell from the left to the right limit quantities. In this way, we cannot use the same technique,

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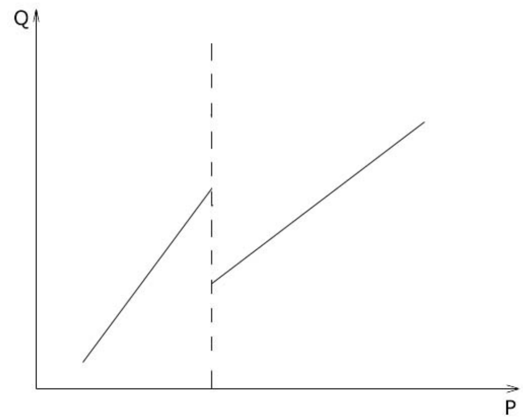
[35] We are taking the independent variable p to be on the horizontal axis.

because the supply function will become decreasing in some interval. For example, consider Figure 3.

Figure 3: Illustration of decreasing supply function in some interval



To cope with this problem we let the first generator reach its maximum capacity in its equilibrium supply function for price p^*. All other generators take part in the game for such price p^* and they do not reach their maximum capacity constraint in their equilibrium supply functions for price p^*.

The residual demand resulting from a price-taking fringe with capacity constraints (D_r) for all generators except the first, satisfy the following condition:

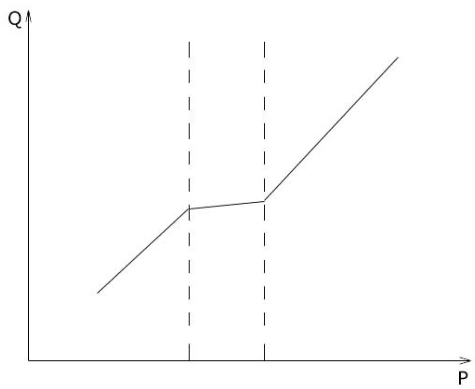
$$-\frac{dD_r}{dp} = \begin{cases} \gamma - \beta_1, p \le p^* \\ \gamma, p > p^* \end{cases} \tag{13}$$

We will make a non-decreasing approximation of the supply function of the j^{th} generator. When $p > p^*$, a supply function $\beta_j'(p - \alpha_j)$ is used. Here β_j' satisfies Equations (8). Prices significantly below p^* are represented by the supply function $\beta_j(p - \alpha_j)$. For prices just below p^* we will rearrange *K-M* equations to obtain slopes β_j'' for yet another affine segment of the supply function so that the supply function is continuous at $p = p^*$. That is, we posit a supply function of the form as in Baldick *et al.* (2000 and 2004):

$$S_j(p) = \begin{cases} \beta_j(p - \alpha_j), p \le p' \\ \beta_j''(p - \alpha_j''), p' < p \le p^* \\ \beta_j'(p - \alpha_j), p^* < p \end{cases} \tag{14}$$

where α_j'' is the solution of the equation: $\beta_j''(p^*-\alpha_j'')=\beta_j'(p^*-\alpha_j)$, so that the supply function is continuous at $p = p^*$ and p' is the solution to: $\beta_j''(p'-\alpha_j'')=\beta_j(p'-\alpha_j)$, so that the supply function is continuous at $p=p'$ as in Baldick *et al.* (2000 and 2004) . The supply function is illustrated in Figure 4.

Figure 4: Supply function



Using the same technique as in Baldick et al (2000 and 2004) we rearrange the *K-M* equations into a standard form of vector differential equation to evaluate the slopes β_j'':

$$\frac{dq}{dp}=\left[\frac{1}{n-2}1\cdot 1^T-I\right]\left[\begin{bmatrix}\dfrac{q_2-x_2}{p-C_2'(q_2)}\\ \\ \dfrac{q_n-x_n}{p-C_n'(q_n)}\end{bmatrix}+\frac{dD_r}{dp}\right]=$$

$$=\left[\frac{1}{n-2}1\cdot 1^T-I\right]\begin{bmatrix}\dfrac{q_2-x_2}{p-C_2'(q_2)}\\ \\ \dfrac{q_n-x_n}{p-C_n'(q_n)}\end{bmatrix}+\frac{1}{n-2}1\cdot\frac{dD_r}{dp} \qquad (15)$$

where q is the supply functions for the n-1 firms[36] and 1 is the column vector of all ones and I is the identity matrix. This is an explicit system of first-order ordinary

[36] All firms except firm 1.

differential equations. Following Baldick et al (2000 and 2004) we evaluate Equation (15) infinitesimally above $p*$ and the infinitesimally below $p*$:

$$\beta''-\beta'=\frac{1}{n-2}(-1\cdot\gamma+1\cdot(\gamma-\beta_1)) \text{ or } \forall j,\beta_j''=\beta_j'-\frac{\beta_1}{n-2}. \quad (16)$$

If any particular candidate's slope β_j'' turns out to be negative, we modify Equation (14) as in Baldick et al (2000 and 2004) into:

$$S_j(p)=\begin{cases}\beta_j(p-\alpha_j), p\le p' \\ \beta_j'(p*-\alpha_j), p'<p\le p* \\ \beta_j'(p-\alpha_j), p*<p\end{cases} \quad (17)$$

As stated in Baldick et al (2000 and 2004) this guarantees that the supply function is non-decreasing.

Baldick *et al.* (2000 and 2004) claim that the general form of this supply function is reasonable in practice even though it may not be an equilibrium under certain conditions. We refer to Baldick et al (2000 and 2004) for further details regarding the derivations and results discussed in this section.

3.5 Building the algorithm in the case of affine marginal costs and maximum capacity constraints

In order to construct the mathematical algorithm in case of affine marginal costs and maximum capacity constraints, the first step is to take the price-axis intercepts of all marginal cost curves of the generators and sort them in increasing order ($a_1 \le a_2 \le ... \le a_n$).

The second step is for each interval $[a_i, a_{i+1})$, (i=1,…,n) to undertake the following steps:

a) We find the number of generators that will take part in the game for such prices.
b) For each generator that will take part and which has not obtained its maximum quantity in the previous interval, we solve the system Equation (8).
c) For each generator a supply function is constructed.
d) For each generator we check whether it obtains its maximum capacity in this interval
d1) If none of the generators obtains its maximum capacity in this interval, we continue with the second step, applied to the next interval.
d2) If there are such generators that reach their maximum quantity in this interval, we determine which one reaches its maximum capacity at the lowest price p_l.

d3) We recalculate again the supply function of the remaining generators in the interval (p_b, a_{i+1}).
d4) We run the procedure again from step d1).

The third step covers the interval $[a_n,+\infty)$ in which we repeat the second step until all generators reach their maximum capacity.

3.6 Supply function equilibrium with piecewise affine marginal cost functions

The model is the same as in section 3-2 except that we now permit each generator's marginal cost function to be a continuous piecewise affine function. We examine the model for which the first generator has a piecewise affine supply function and the remaining generators have an affine marginal cost function. The first generator's marginal cost function breaks at a capacity q^* and a price p^* less than α_j and does not have range restrictions.

$$C_1'(q) = \begin{cases} c_1^1 q + a_1^1, q \le q^* \\ c_1^2 q + a_1^2, q > q^* \end{cases} \tag{18}$$

$$C_i'(q) = c_i q + a_i, \quad \forall i \ne 1$$

The set of differential equations that solves the slopes of supply functions of the generators in the two segments for $q \le q^*$ and for $q > q^*$ are:

$$\beta_1 = \frac{\gamma + \sum_{i \ne 1} \beta_i}{1 + c_1^1(\gamma + \sum_{i \ne 1} \beta_i)} \tag{19}$$

$$\beta_i = \frac{\gamma + \sum_{j \ne i} \beta_j}{1 + c_i(\gamma + \sum_{j \ne i} \beta_j)}$$

and

$$\overline{\beta}_1 = \frac{\gamma + \sum_{i \ne 1} \beta_i}{1 + c_1^2(\gamma + \sum_{i \ne 1} \beta_i)} \tag{20}$$

$$\overline{\beta}_i = \frac{\gamma + \sum_{j \ne i} \beta_j}{1 + c_i(\gamma + \sum_{j \ne i} \beta_j)}$$

We solve both these system of equations and find two different families of solutions. The supply function of each generator has discontinuous slopes at the point q^*, left-hand slope β_j' and right-hand slope β_j''. Similarly the respective intercepts are α_j and α_j''. So we have a pair of supply functions for the i-th generator:

$$q = \beta_i'(p - \alpha_i), p \le p*$$
$$q = \beta_i''(p - \alpha_i''), p > p*$$

The possible further constructions of the supply functions are:

Step 1: In case $\beta_i'(p^* - \alpha_i) < \beta_i''(p^* - \alpha_i'')$ we assume that the generator can supply all possible quantities between $\beta_i'(p^* - \alpha_i)$ and $\beta_i''(p^* - \alpha_i'')$ for the price p^*.

Step 2: In case $\beta_i'(p^* - \alpha_i) \ge \beta_i''(p^* - \alpha_i'')$ we let p' be the solution of the equation:
$q* = \beta_i''(p' - \alpha_i'')$. Then the supply function becomes:

$$q = \beta_i'(p - \alpha_i), p \le p*$$
$$q = q^*, p^* < p \le p',$$
$$q = \beta_i''(p - \alpha_i''), p > p'$$

The supply curves of the other generators will have different slopes but the same intercepts for quantities below q^* and above q^*. That means that this function is not continuous. Also, for the case of n–generators, it is possible to find the SFE under these assumptions via the following procedure. There are two different variants of generator's 1 supply curve that occur as shown in Figures 5 and 6.

Figure 5: Illustration of discontinuous supply function

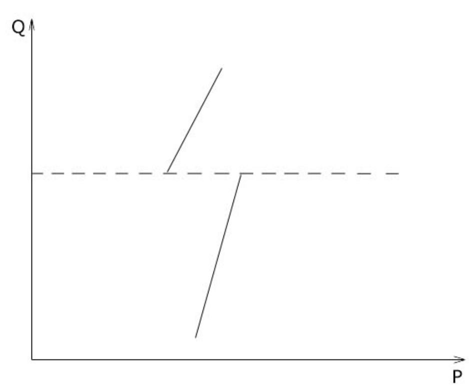
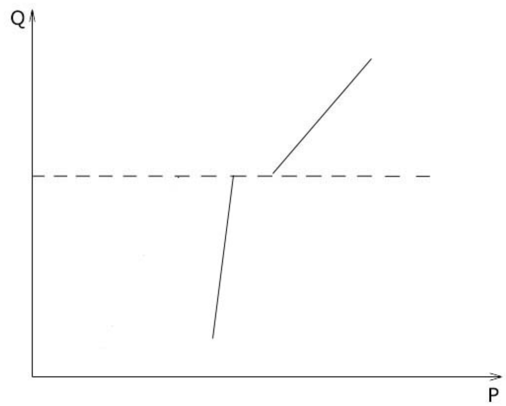


Figure 6: Illustration of discontinuous supply function

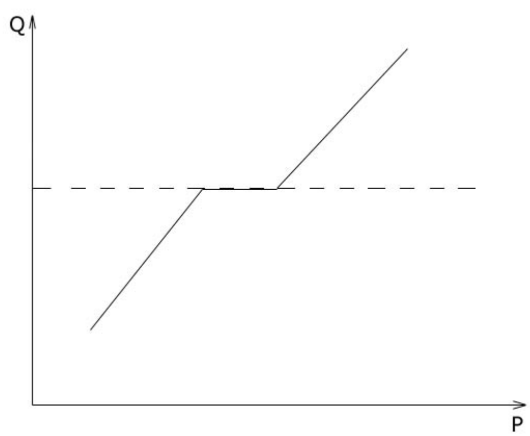


So the major problem that occurs in case of piecewise affine marginal costs, is how to adjust these two curves to obtain a continuous function. Below we discuss how we solve this problem.

Let the upper curve intercept q=q*, at p= p^U , and the lower curve intercept the lower curve at p= p^l . In this case two variants occur:

1) Let $p^U > p^l$ as shown in Figure 6, then the supply function is not defined for values between p^l and p^U . The supply function must be non-decreasing so the only way to extend this function between these two points, without changing the function from the left of p^l and to the right of p^U , is to define: $q(p) = q*$, for $p^l <$ p $< p^U$. It is non-optimal because it does not satisfy the *K-M* differential equations, but it is the only extension between two optimal lines. We call this a first-order extension and it is illustrated in Figure 7.

Figure 7: Illustration of a first-order extension



2) Let $p^l > p^U$ as shown in Figure 5, then there are two optimal supply curves in interval $p \in (p^l, p^U)$. So there does not exist a unique supply function equilibrium in this interval. Every equilibrium is an approximation between these two curves. What the real equilibrium (i.e., the equilibrium selected via this algorithm) is, depends on how the learning process is applied. We will make the following assumptions regarding the learning process[37] needed to select a unique supply function equilibrium:

a) We assume that at the end of the bidding day every generator knows the aggregate supply function of the remaining generators (with the help of some regression methods).
b) Each generator uses its marginal cost curve as its first bid.
c) Each generator assumes that other generators hold their own supply curves constant.
d) Each generator constructs its supply function following these steps:
d1) if its marginal cost function is affine, it constructs the functions as described in the previous sections.
d2) if its marginal cost function is piecewise affine then the generator will construct its curve as follows:

Constructs supply function S_1, based on the first (lowest) segment of its marginal cost function.

Finds the price at which the supply function reaches the maximum capacity of this segment of the marginal cost function $\equiv p_1$.

Constructs supply function S_2 based on the next segment of the marginal cost function. Then finds the price at which the minimum quantity of this segment is achieved $\equiv p_2$.

If $p_2 > p_1$ then the generator will make a first-order extension as in paragraph 1) above between prices p_1 and p_2

If $p_1 < p_2$ then the generator constructs the supply function on the following manner: for prices bellow p_1 it will submit S_1; for prices over p_1 it will submit S_2; for price p_1 it will submit all the quantities between $S_1(p_1)$ and $S_2(p_1)$ and so on for all segments of the marginal cost curve as shown in Figure 8.

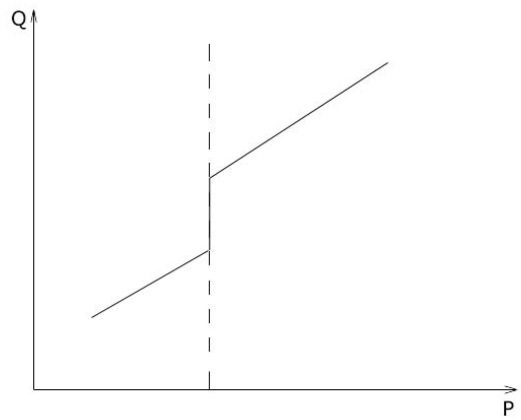
Under these assumptions, the equilibrium supply functions are characterized as follows:

- For prices less than p^l, the equilibrium supply function will be the lower supply function,
- For prices greater than p^l, the equilibrium supply function will be the upper function, and
- For the price p^l, the generator will submit all quantities between the values of the upper and lower supply functions in p^l.

[37] The learning process shows the required number of days for equilibrium to be achieved in each demand segment. In large demand segments (with a large number of days included) the required number of days for equilibrium to be achieved is negligibly small and therefore may be omitted. Rudkevich (1999) proved convergence if all firms initially bid competitively. Baldick et al (2000) showed conditions for the update to be a contraction map, implying that convergence would occur whether or not the initially bids were competitive.

We will refer to this as the second-order extension as shown in Figure 8. With this we can generalize the algorithm for piecewise affine marginal costs.

Figure 8: Illustration of a second-order extension



3.7 Building the algorithm in the case of piecewise affine marginal costs

We follow the same algorithm as outlined in Section 3-5 with some additional steps described below. In each iteration, in addition to checking whether any of the generators reached their maximum capacity, we also check if any of them reaches a quantity for which the marginal cost curve changes its slope. If there are no such generators, the known procedure of Section 3-5 applied. If there are such generators, we recalculate new supply functions for the new smaller interval and run the same procedure for this interval. The algorithm stops when all generators reach their maximum capacities. At the break points of the supply functions we define the supply function so that it is continuous, as explained in the previous sections.

4. Application to the Dutch wholesale market

4.1 Overview of the Dutch market

In the Netherlands, the liberalization of the electricity industry started with the Electricity Act of 1998 implementing the European Electricity Directive 96/92. The main lines of the liberalization process consisted of a gradual opening of the electricity market and a complete unbundling of the generation and supply of electricity from the transmission business. From a market design point of view, four different types of markets need to be considered. First, one of the most visible aspects of the liberalization process was the establishment of a voluntary day-ahead market, the Amsterdam Power Exchange (APX). Second, in parallel to the power exchange, a bilateral market exists where buyers and sellers arrange bilateral contracts and then notify the transmission system operator (TenneT) about their intended level of production and consumption. Third, a balancing market run by the system operator was established to price deviations between both actual production and consumption and the day-ahead plan. Finally, an important feature of the Dutch market since 2001 has been the use of a separate auction for cross-border capacity.[38] Total consumption in the Netherlands is about 100 TWh. The 650 largest industrial consumers (33% of total demand) were granted access to the market in 1998. The second step of market opening (33%) was accomplished in January 2002. The total installed generation capacity is about 20 GW. The existence of a few dominant generators, a large share of decentralized units, and important cross-border flows characterize the Dutch electricity market. Gas fired power plants represent the vast majority of the technology used, followed by coal fired plants. Four generators own roughly 60% of the installed capacity: Essent/EPZ, Electrabel, Reliant/Nuon and E.On. At the time of market opening more than 40 regional supply and distribution companies existed.

The Transmission System Operator (TenneT) plays a key role in the operation of the power market. TenneT is responsible for the national power grid (380kV and 220kV) and for the interconnectors with Germany and Belgium. TenneT also is responsible for real-time balancing. Since January 2001, TenneT operates a "balancing market", i.e., a market for regulation and reserve power that is used to cope with real-time imbalances and congestion within the Dutch system. Market participants can submit bids for increasing or decreasing generation from their DAM schedules.

APX started its operations in May 1999 with a voluntary day ahead spot market for electricity. The APX offers hourly contracts one day ahead of delivery and the price is based on a two-sided market clearing or auction mechanism of the aggregated curves of all offers and bids. The trading on the APX started on 25 May 1999 and amounted to some 2% - 3% of the Dutch national electricity consumption. The principal goal of APX is to facilitate a spot market or day-ahead market ("DAM"). A short-term market was considered as a necessary first step for the establishment of a derivatives and financial market. The APX was the first electricity exchange trading internationally as several foreign participants joined the APX and both imported and exported electricity was traded via APX. Such international trade has been facilitated by the fact that some inter-connector capacity was reserved to the APX.

Since the start of the Dutch power exchange, price-spikes above 1000 Euro/MWh have been regularly observed and have caused a heated debate on the correct functioning of the market. It has been argued that these price-spikes were caused by coincidental reduction of supply because of maintenance and unavailability of power plants in the Netherlands and Belgium but allegation have been made that dominant generators deliberately withdrew generation capacity and offered higher prices on the APX. Table 1 provides an overview of the average, standard deviation, minimum and maximum prices at APX since its beginning until the end of 2004. The average price (volume-weighted) has ranged from 27.34 (Euro/MWh) in 1999 to 52.85 (Euro/MWh) in 2000. The price was most volatile in 2003 with a standard deviation that was about twice as large as the absolute value of the price. The standard deviation is also relatively large for the other years. The

[38] It is worth noting that there exist a direct relationship between the interconnector auction and the APX since players who have obtained interconnector capacity at the daily auction are forced to offer it to the APX.

lowest minimum price observed is 0.01 (Euro/MWh) 2002-2004, while the highest observed price was 2000 Euro/MWh in 2003. In our simulations we have chosen 2003 as the reference year.

Table 1: APX day-ahead prices from 1999 to 2004

APX prices (Euro/MWh)						
year	1999	2000	2001	2002	2003	2004
average	27.34	52.85	33.41	27.64	45.25	31.35
standard deviation	8.85	68.63	56.31	41.45	101.44	22.26
min	14.62	18.00	0.01	0.01	0.01	0.01
max	50.00	572.00	1600.00	701.00	2000.00	800.00

Imports play an important role in the Dutch power market and have doubled from 10% to 20% of total demand after the opening of the market. As market prices in the Netherlands have been higher than in neighbouring countries, market participants have put enormous pressure on TenneT to make cross-border transmission capacity available to the market. Since January 2001, TenneT, in cooperation with the neighbouring TSOs, has organized auctions to sell cross-border transmission capacity. The capacity is allocated for the different interconnectors (borders) in yearly, monthly and daily auctions. Participants can submit bids (for price and volume) with a maximum volume per participant of 400 MW. The selling price is determined by the marginal bid (lowest bid price to which capacity is allocated). The auction is run completely independent from APX, although participants that acquire import capacity at the daily auction are obliged to use this capacity to sell in the APX spot market.

Finally, the Dutch energy regulator DTe supervises the compliance with the Electricity Act. The DTe issues supply licenses for the supply of electricity and gas to captive customers, determines the tariff structures and conditions for the transmission of electricity, and determines connection, transmission and supply tariffs for electricity which used "price cap" regulation aimed at promoting the efficient operation of the electricity grid and gas networks.

4.2 Modeling assumptions and parameters

In this section we elaborate on the assumptions and the parameters used for applying the Symbad model to the Dutch electricity market. This description illustrates the importance of assumptions in market modeling and the flexibility of the SFE approach in incorporating real-world market features. Several essential parameters for the Dutch market have been defined:

➢ ***Import-export:*** because the Dutch market is heavily dependent on import reasonable assumptions regarding the volumes of electricity imported and exported have been made. These volumes have been separated into two categories. On one hand, one part of the volumes has been attributed to the main Dutch electricity producers. An allocation has been assumed for the import capacity among different large players based. Therefore out of the 3350 MW of available capacity for import, 1070 MW have been allocated to the four large player while the rest has been allocated to the fringe. For simplification, the marginal cost of imports for the two categories was assumed to be at the lower end of the Dutch plants, i.e., imports would always be competitive.

➢ ***Demand elasticity:*** all market simulation models are heavily dependent on the assumptions made regarding the elasticity of demand. In accordance with best practice we used a demand elasticity of 0,1. To study of the impact of changes in the elasticity we undertook a sensitivity analysis with different value for the elasticity.

Table 2: Assumptions regarding load

		hours	load MW
Winter	superpeak	50	14701
	peak	704	13949
	shoulder	704	11527
	offpeak	704	9234
Midseason	superpeak	100	13543
	peak	1429	12652
	shoulder	1429	11007
	offpeak	1429	8763
Summer	superpeak	50	12722
	peak	720	12421
	shoulder	720	10590
	offpeak	720	8841

➢ ***Demand levels:*** three seasonal levels of demand were considered: winter, midseason and summer which again were further differentiated into superpeak, peak, shoulder and offpeak levels. We assume a corresponding duration of each level as shown in Table 3. The load varies from 8763 MW to 14701 MW and we have corrected it for small scale CHP loads. These levels of demand and duration were used to calculate an annual volume weighted price. The SYMBAD model provides equilibrium quantities of demand and prices. The simulated quantities may not match exactly the actual demand in the Dutch market. Therefore we establish linear regression relationships so that the corresponding prices to the levels of demand in Table 2 could be calculated.

➢ ***Import restrictions:*** according to Dutch regulations market parties in the Netherlands are not allowed to own more than 400 MW of interconnector import capacity. Such measure was designed to mitigate market power. This characteristic is considered in the model.

➢ ***Generator availability:***[39] we assumed that generating assets have availability in the range 86%-95% reflecting planned and unplanned outages. This figure

[39] The modeling of availability of units is treated as available prorated capacity in all hours.

represents an average figure based on classical technical figures and experience gained from a range of assignments previously undertaken by the authors.

- ***Power plant technical data:*** the portfolio owner, maximum generation capacity, operating and maintenance costs, heat rates, and fuel prices were specified. Maximum plant generation capacities ranged from 6 MW to 658 MW and the total generation capacity (excluding import) was 20858.4 MW. The market shares of the four main players are Essent (24.4%), Electrabel (24.1%) Nuon (19.6%) and E.ON Benelux (9.0%).

- ***Power plant variable costs:*** the total variable costs ranged from 12.7 Euro/MWh to 125.8 Euro/MWh. In this modeling exercise fixed costs were not considered. Therefore the units are assumed to be online in every hour regardless of their economics unless they incur a forced or maintenance outage, so that unit commitment costs are ignored. Minimum load blocks are also ignored. Reserve requirements are not considered. The model does not distinct between day-ahead and real-time variable costs. Indeed such a distinction would require considering unit commitment costs. In summary we assumed (1) linear marginal cost curves estimated based on engineering data, (2) all units not unavailable due to forced or maintenance outages are available, and (3) the market is simulated without operating reserves.

- ***The level of forward contracting:*** In four scenarios no forward contracting is considered while in the last scenario 10% of generation capacity is contracted forward. This number is chosen for computational convenience. Higher levels would cause convergence problems. The real level of forward contracting in the Dutch market is higher but any precise number is difficult to specify as the majority of contracts are traded bilaterally and therefore not public information is available. On this issue most European electricity markets are very different to most markets that can be found worldwide. Indeed most electricity markets worldwide have been designed around a mandatory organized market place (i.e. pool) for spot physical transaction completed with bilateral financial contracts. In Europe most markets are organized around physical bilateral transaction completed with a voluntary organized market place (i.e. exchange).

- ***Strategic behaviour:*** All firms are supposed to act strategically. Table 5 provides an overview of the different scenario (S2 to S5) compared to the base case scenario (S1). With the exception of scenario S1, the firms are assumed to behave strategically. In scenario S1 we assumed that players do not behave strategically. Therefore this base scenario represented a competitive benchmark (price equals marginal cost). In a second step we assumed strategic behaviour by market participants in scenarios S2 to S5. For this purpose we have used the supply function equilibrium concept to quantify the impact of strategic behaviour on market output. With Scenario S1, S2 will be used as a counterfactual against which the scenarios S3 and S4 (elasticity 0.01 and 0.05 respectively) and scenario S5 (10 % of generation capacity forward contracted) can be compared.

Table 3: Scenario overview.

	S1	S2	S3	S4	S5
Elasticity	0.1	0,1	0.01	0.05	0.1
Import	Yes	Yes	Yes	Yes	Yes
Strategic Behaviour	No	Yes	Yes	Yes	Yes
Availability	86% to 95%	86% to 95%	86% to 95%	86% to 95%	86% to 95%
Forward contracts	No	No	No	No	10%

Figure 9: Merit order curve in the Netherlands

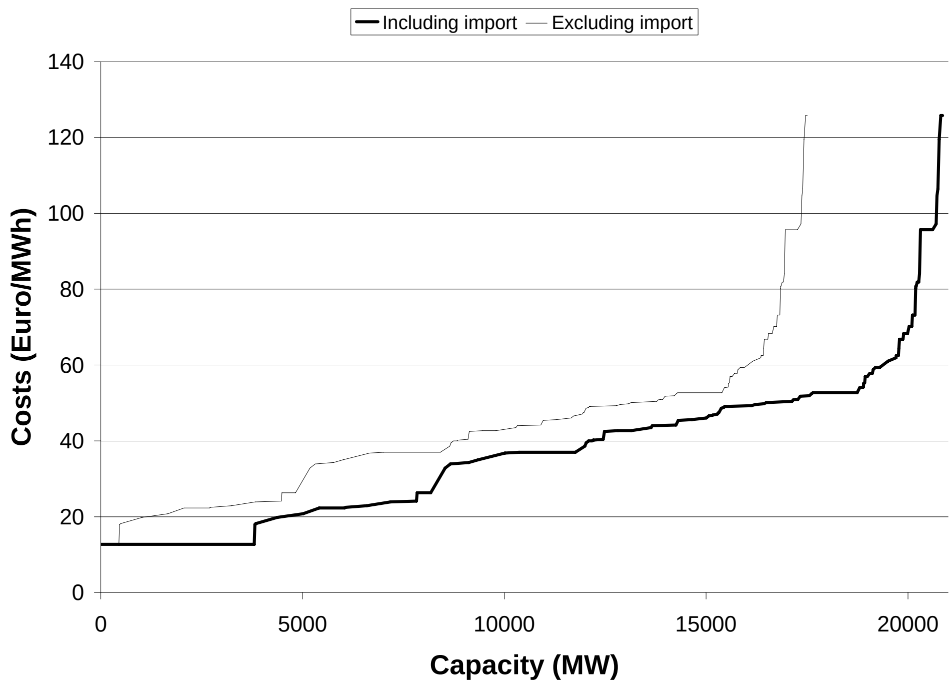


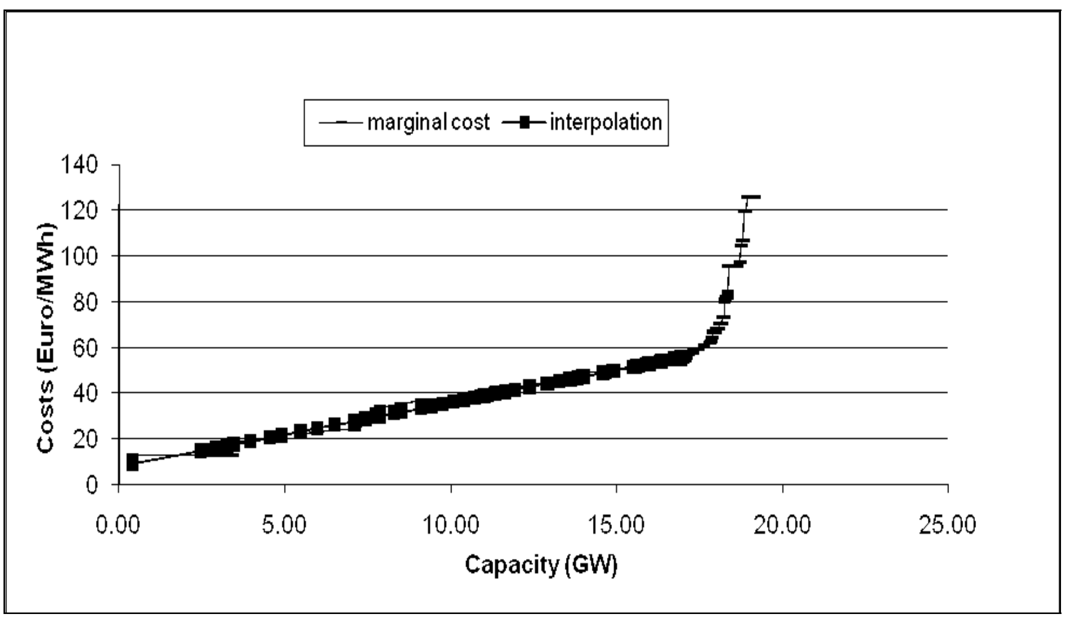
Figure 9 presents the actual marginal costs. Two merit order curves are presented. The first one includes imports while the other one includes only domestic generation. For simulations with strategic behaviour, SYMBAD provides the equilibrium quantity and equilibrium price and derives a separate set of mark-ups for each demand segment and for each value of demand.[40]

[40] The competitive price can be estimated "backward" by subtracting the mark-up (expressed as a percentage of the competitive price) to the new equilibrium price. SMPMC= SMPSFE- Mup, where SMPMC is the equilibrium price on perfectly competitive market, SMPSFE is the equilibrium price in a market with strategic behaviour and MUp is the mark up. Differences between the perfectly competitive price and actual marginal costs are due to the fact that SYMBAD estimates marginal costs using affine or piecewise affine curves in order to be modeled according to the linear SFE concept.

4.3 Results of the simulations

In this section we use the models described to simulate the potential impact of strategic behaviour on electricity prices using the SFE framework as applied in the model. Our objective is to illustrate how strategic behaviour, elasticity values and forward contracts affect the equilibrium quantities and prices and how careful calibration can improve the results of the model in order to fit actual market outcomes. For this purpose we performed several simulations for the Dutch market. For the different scenario, the equilibrium quantity and price in a perfectly competitive market and in the market under strategic behaviour are defined as the intersection between the demand curve and the marginal cost curve (or the supply function curve in the case of strategic behaviour). The model derives a separate set of mark-ups for each demand segment and for each value of demand within the separate demand segments. The price mark-up is defined as MUp = MPSFE – MPMC, where MPSFE is the equilibrium price in the market with strategic behaviour and MPMC is the equilibrium price in the perfectly competitive market. The model is also able to determine supply mark-up. While price mark-ups are relative to the perfectly competitive market price, supply mark-ups are relative to the equilibrium price of demand and supply, where the demand is the demand in the new equilibrium. The demand in the new equilibrium is lower than the demand in the perfectly competitive equilibrium because higher prices cause customers to consume less; i.e., demand elasticity. Both price and supply mark-ups can be used as indicators of market power, but price mark-ups[41] are normally used in studies such as these because the reference price is clearer.

Figure 10: Merit order curve in the Netherlands corrected for unavailability



41 Prices are expressed in percentage of the competitive price.

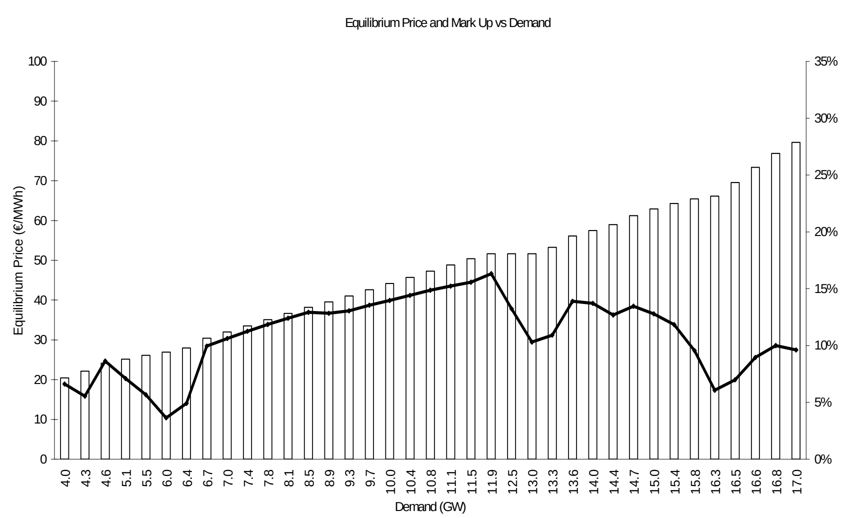
4.3.1. *Scenario S1*

In this scenario the merit order curve is corrected for plant unavailability (Figure 10). Based on this we created a linear interpolation of the curve in the range 0-17 GW. Next we determined the corresponding price to each of the load levels in Table 2 and calculated a volume weighted price for each the seasons: winter, midseason and summer as well as the full year. The annual volume weighted price was 41.64 Euro/MWh, which is roughly 10 % below the actual average price observed in 2003 (45.25 Euro/MWh).

4.3.2. *Scenario S2*

In contrast to scenario S1 this scenario demonstrates the impact of strategic behaviour. Figure 11 shows equilibrium prices and price mark-ups as a function of demand for scenario S2. The results show an important increase in the level of the price mark-up with the level of demand. The price mark-up exceeds 10 % for demand levels above 6.7 GW and reaches 21.7% for a demand level of 11.5 GW. One can clearly see that the mark-ups do only slowly increase with increasing demand; in fact, they may even decrease again[42]. At a certain point, mark-ups suddenly start rising, indicating substantial market power of some generators. This phenomenon is usually explained by the fact that most participants have already bid in all of their capacity and the remaining generators (i.e., generators that still have available capacity) can exercise significant market power. This effect may lead to extreme mark-ups in markets with dominant firms. The main point here is that even small, supposedly negligible fluctuations of demand may thus lead to extreme price variations. Also for this scenario we created a linear regression between equilibrium prices and quantities. This relationship was then used to calculate volume weighted prices in the same manner as for scenario S1. The volume weighted price was 48.85 Euro/MWh, which is 8% above the actual value for 2003 (45.25 Euro/MWh).

Figure 11: Price mark-ups and strategic price as function of demand for scenario S2



[42] These drops in the price-mark-ups can be explained by the fact that in some demand segments the degree of competition may rise (larger number of players starts competing), such that prices increase more slowly than marginal cost.

4.3.3. Scenario S3

This scenario is similar to scenario S2, but now the elasticity of demand is 0.01 compared to 0.1 for scenario S2. We discuss the impact on the volume weighted price in section 4-3-6 where we make a comparison between all the scenarios. The annual volume weighted price was 50.50 Euro/MWh. Figure 12 shows the prices and price mark-ups as function of demand. The price increases with increasing demand and reaches a maximum of around 615 Euro/MWh for a demand of 18.9 GW. The mark-up exhibits a top around 110% for a demand of around 17.5 GW.
Figure 12: Price mark-ups and strategic price as function of demand for scenario S3.

4.3.4. Scenario S4

This scenario is similar to scenario S3, but now the elasticity of demand is 0.05 compared to 0.1 for scenario S2. Also here we discuss the impact on the volume weighted price in section 4-3-6 where we make a comparison between all the scenarios. The annual volume weighted price was 49.37 Euro/MWh. Figure 13 shows the prices and mark-ups for scenario S4. Also here the price increases with increasing demand. The mark-ups are high for some levels of demand (for example 6.6 GW to 11.3 GW) and have some "dips" for a level of demand around 6.2 GW, 12.9 GW and 16.3 GW.

Figure 12: Price mark-ups and strategic price as function of demand for scenario S3

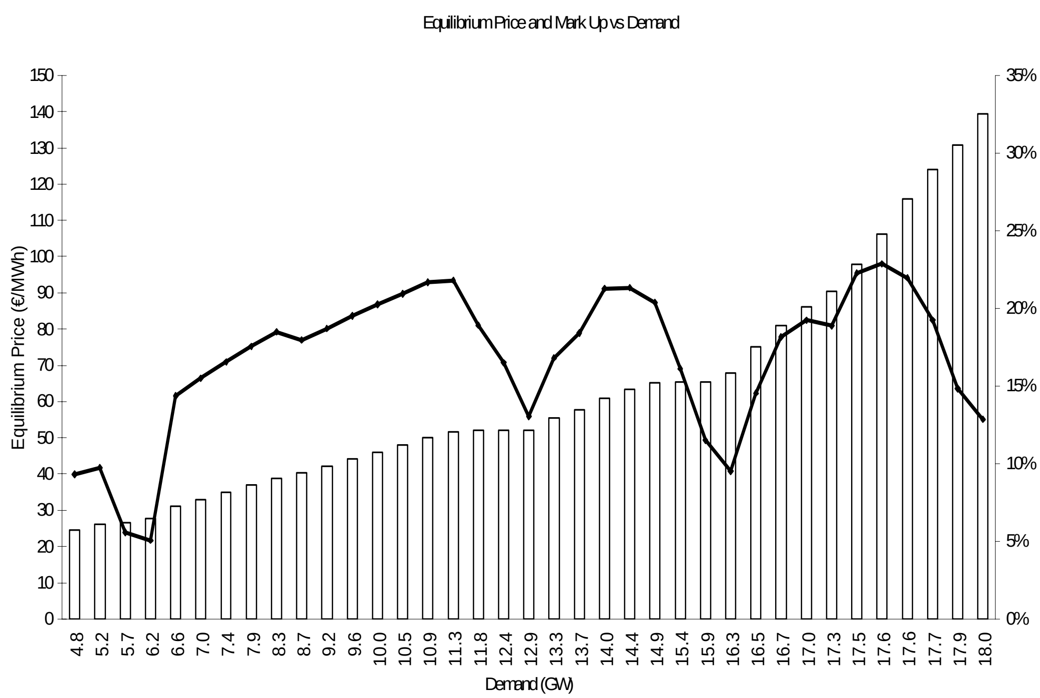
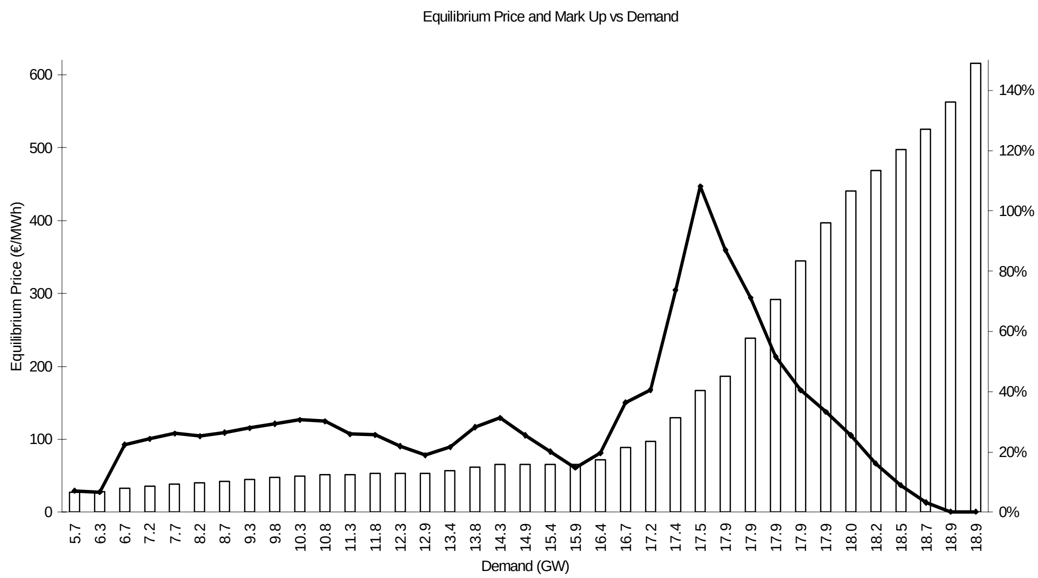


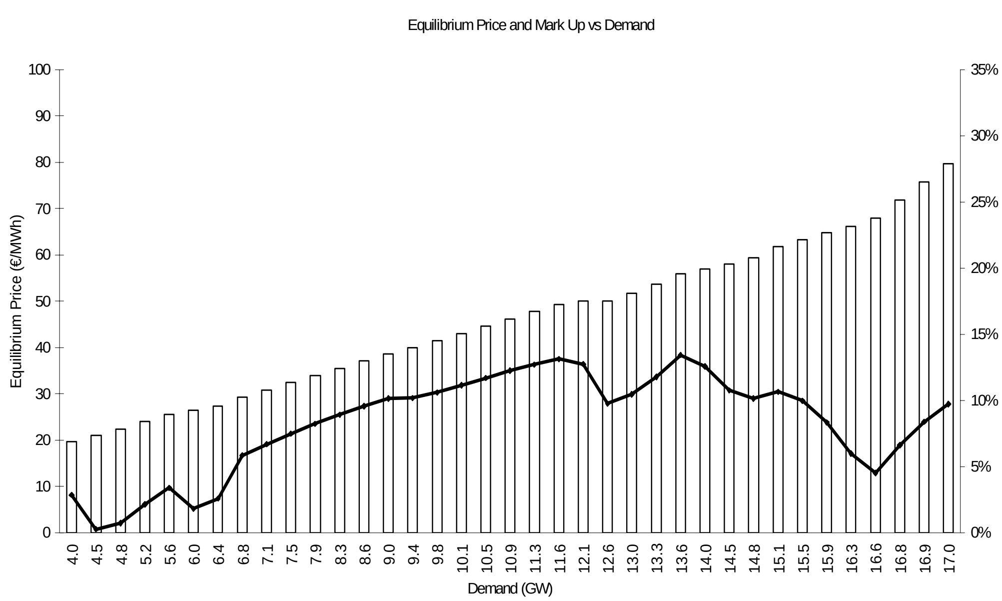
Figure 13: Price mark-ups and strategic price as function of demand for scenario S4



4.3.5. Scenario S5

This scenario is similar to scenario S2 but with the difference that 10% of the portfolio owners' output are contracted forward. The annual volume weighted price is 46.77 Euro/MWh. Figure 14 shows the prices and mark-ups for scenario S5. The price increases with increasing demand. The shape of the mark-up curve is similar to scenario S2 but the mark-ups are generally lower.

Figure 14: Price mark-ups and strategic price as function of demand for scenario S5



4.3.6. Price comparison of the scenarios

In order to compare the different scenario, a linear regression between the price and demand for all scenarios was performed. Based on these regressions we could a find a corresponding price for each demand levels in Table 2 as shown in Table 4.

Table 4: Corresponding prices (Euro/MWh) for each load level for all the scenarios

		hours	load MW	S1	S2	S3	S4	S5
Winter	superpeak	50	14701	49.27	62.46	58.63	62.56	59.68
	peak	704	13949	47.01	59.30	58.63	59.49	56.68
	shoulder	704	11527	40.24	49.81	51.72	50.30	47.68
	offpeak	704	9234	33.76	40.71	42.89	41.49	39.05
Midseason	superpeak	100	13543	45.88	57.72	58.63	57.96	55.18
	peak	1429	12652	43.49	54.36	56.13	54.71	51.99
	shoulder	1429	11007	38.83	47.83	49.80	48.39	45.80
	offpeak	1429	8763	32.49	38.93	41.16	39.77	37.36
Summer	superpeak	50	12722	43.63	54.55	56.32	54.90	52.18
	peak	720	12421	42.78	53.37	55.17	53.75	51.05
	shoulder	720	10590	37.56	46.05	48.07	46.66	44.12
	offpeak	720	8841	32.63	39.13	41.35	39.96	37.55

Furthermore we calculated seasonal and annual volume weighted price as shown in Table 5. We observe that the impact of strategic behaviour on the annual volume weighted prices is significant, the price increase by 17.3%. Furthermore when the demand elasticity is decreased to 0.01 (scenario S3), the price increased by 1.65 Euro/MWh. For an elasticity of 0.05 (scenario S4) the price increase is less, 0.52 Euro/MWh. Finally the price decrease by 2.08 Euro/MWh when 10% of the generator output is forward contracted (scenario S5). The price levels in 7 Table generally fit the price level observed at APX in 2003 (45.25 Euro/MWh). Figure 15 shows the price duration curves for the APX day-ahead price, the competitive price and the SYMBAD scenario (S2) price. The APX day-ahead price curve was constructed by sorting the hourly prices in 2003 by decreasing order. The competitive price curve was calculated by using the corresponding price to 29 separate load levels. The price curve the SYMBAD S2 scenario was constructed in the same manner. It can be observed that the APX prices exhibit a higher price level than the competitive price for the first 1900 hours of 2003. After this the competitive price is higher than the APX price. The SYMBAD S2 price is lower than the APX price for the first 1200 hours of 2003 and the higher the remaining hours of the year. The volume weighed average APX price was 45.25 Euro/MWh in 2003, while the competitive price is 41.64 Euro/MWh and the SYMBAD S2 price is 48.45 Euro/MWh. Hence there appears to be a good correspondence with market price observed in the Dutch market. It is seen that the deviation between the APX price and the other prices is largest for the first hours, especially peak load hours. However it is worth noting that we have based our calculations for the competitive price and the S2 price on a single average price for each characteristic load level.

Table 5: Seasonal and annual volume weighted prices (Euro/MWh)

	S1	S2	S3	S4	S5
Winter	41.47	51.53	52.33	51.97	49.31
Midseason	39.13	48.25	50.18	48.79	46.20
Summer	46.89	47.19	49.17	47.76	45.19
Annual	**41.64**	**48.85**	**50.50**	**49.37**	**46.77**

Figure 15: Price duration curves for APX, merit order and SYMBAD S2 scenario

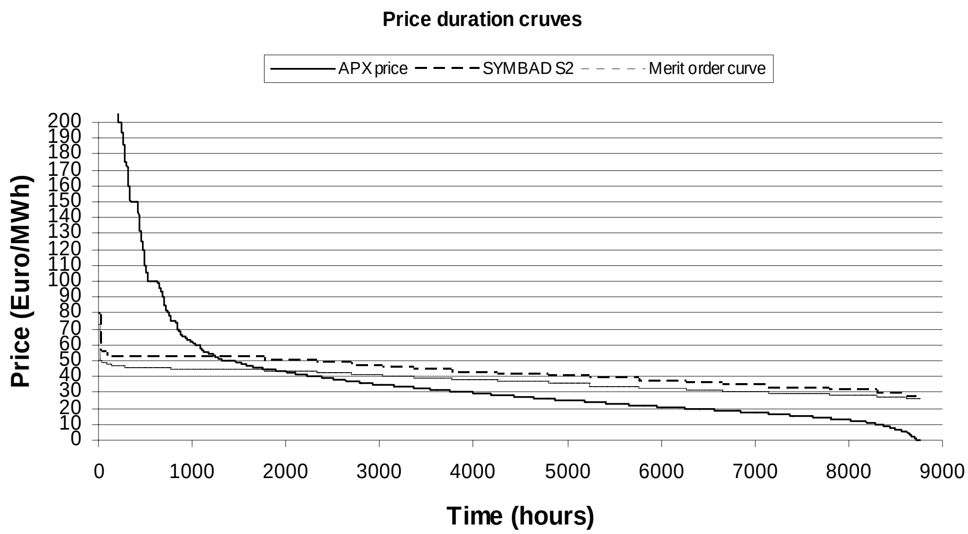
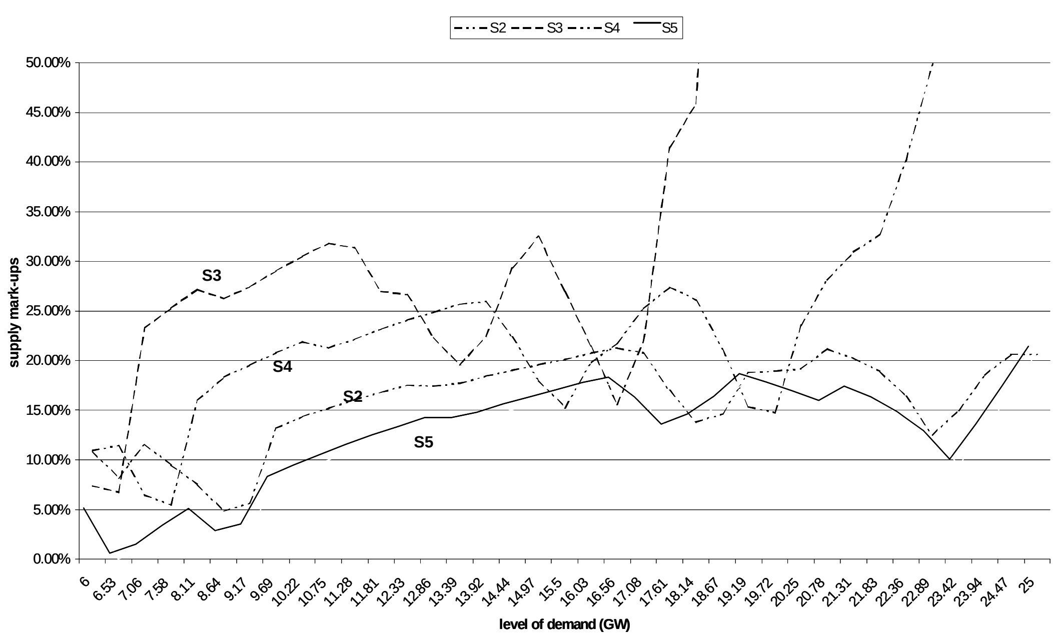


Figure 16 shows the supply mark-ups as function of similar equilibrium quantities. This allows for a comparison between different supply mark-ups. The supply mark-ups decrease when 10% of the generator output is forward contracted (scenario S5). Conversely when the elasticity is changed to 0.01 (scenario S3) the mark-ups increase. The same trend is observed for an elasticity of 0.05 (scenario S4) but not to the same extent as for scenario S3.

Figure 16: Comparison of price mark-ups as function of demand level.



Of course, these results have to be viewed with some caution since they are based on a highly simplified model of the Dutch power market. However, such a model may be used for exploring alternative options and assessing different scenarios, keeping in mind that the main danger lies in misinterpretation and inferring too much precision in quantitative results. For instance, following careful calibration such model may be used to estimate the potential impact of a takeover of one generator by another, of the construction of new interconnector capacity, or a change in market rules.

5. CONCLUSIONS

In conclusion, research into modeling electricity markets is continuing and is the subject of many debates. All types of competition (Cournot, Bertrand, supply function) are utilized and have their advantages and disadvantages for electricity markets. It is well-recognized that models cannot address all questions of interest; however, they appear as an interesting tool for gleaning insights into the complexity of electricity markets and whether electricity markets may deliver the expected benefits of liberalization. In particular, proving the existence of market power is a very complex task. Market simulation models should not be seen as the ultimate solution but as one powerful tool. While it is extremely difficult to prove if any market participants were manipulating markets, simulation models can show (under certain assumptions) if it would have been profitable to do so. Such models can be used in addition to traditional competition analysis. For instance, a model can estimate different benchmarks (competitive, supply function) against which actual market prices may be compared.

In the application of the SYMBAD model to the Dutch market we found that the model calculated reasonable prices. For example the volume weighted Dutch APX was 45.25 Euro/MWh in 2003. On the other hand, the competitive price was 41.64 Euro/MWh and the strategic behaviour scenario S2 price using SYMBAD with an elasticity of demand equal to 0.1 was 48.45 Euro/MWh. Hence the competitive price is approximately 8% lower than the APX price while the SYMBAD price is approximately 7% higher. Therefore the observed APX price appears to in the middle of the competitive and strategic case. When the price duration curves for the three cases are constructed it is observed the APX prices exhibit higher price levels during peak load hours while the competitive and strategic cases exhibit higher price levels during mid-and baseload hours. This can be explained by two important features that cannot be captured by the model. First, during the off-peak period, the APX price regularly reaches a level of 0.01 which is well below any actual marginal cost. This is due to the fact that during off-peak periods several generators are offering their capacity at that minimum price since they are ramping up their power plants and these additional outputs could not be contracted bilaterally. For instance a power plant that is planned to produce 50 MWh from 8.00 a.m. will already be producing a lower quantity before, this quantity can be offered on the power exchange. Second very high prices can be explained by the fact that the model does not consider fixed costs. However since some power plants are only running for very few hours during the year they would not be able to cover their costs by offering their short run marginal costs. Additionally, the fact that supply curves on the APX are discontinuous (or "step curves") compared to continuous supply curves used in the models implies that even assuming perfectly competitive conditions prices may differ from the marginal cost of the last unit produced. Finally the calculations of the competitive and strategic price duration curves are affected by the fact that we selected characteristic load levels and corresponding prices. Hence the models cannot capture some of the hourly resolution found in the APX prices. Decreasing the elasticity of demand to 0.01 (scenario S3) resulted in a price increase of 1.65 Euro/MWh compared to S2. For an elasticity of demand of 0.05 (scenario S4) the price increase compared to S2 was 0.52 Euro/MWh. Finally for an elasticity of demand equal to 0.1 and 10% of the generator output contracted the price decrease compared to S2 was 2.08 Euro/MWh.

The analysis presented in this paper provides a first step in characterizing and quantifying electricity-pricing behaviour using the supply function approach combined with forward contracts for the Dutch electricity market. When keeping in mind that the main danger lies in misinterpretation and inferring too much precision in quantitative results such modeling efforts are of interest for regulatory authorities and policy makers in providing quantitative measures when for instance (1) assessing whether a market participant is becoming dominant and can abuse its market power, (2) considering any measures related to the improvement of market rules and (3) strengthening market monitoring. In this way, the modeling approach may be applied to assess the impact of regulatory or political interventions (such as price caps, incentives for particular generation technologies or fuel diversity, and preferential dispatch) on market performance. Similarly, such models provide an additional tool that can be used by industry stakeholders to (1) forecast market price developments, (2) estimate revenue streams for asset valuation purposes, (3) understand the rationale and drivers of bidding strategies, and (4) evaluate/optimize power supply portfolios.

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