RISK MANAGEMENT IN CONGESTED ELECTRICITY NETWORKS

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ABSTRACT

In this paper we describe available instruments for hedging against transmission congestion risks. These include forward contracts and options. We illustrate risk management strategies for trades between two locations when transmission congestion is present. Risk management in three different markets is exemplified by the general forward market, the bilateral market, and the Nordic market. Cash flow analysis describes the conditions under which hedging is profitable and demonstrates that players can protect themselves against future price differences. Taking into account that a riskless hedge may be non-optimal if the objective is to minimize variance, the optimal hedge ratio for forward contracts is calculated.

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INTRODUCTION

After deregulation of electricity markets, price volatility has increased (see for example Energy Information Administration, 2002). Moving from regulated markets with no or very low price uncertainty, the electricity markets are now facing liberalization and restructuring. Electricity prices are no longer determined by the regulator, but by the market. Experience from the Nordic and California electricity markets demonstrates that the prices may exhibit extreme volatility. Therefore, hedging instruments play an important role in the most well-functioning markets (e.g. Nord Pool). Trading across different regions creates risks that can be managed by use of financial transmission rights (Hogan, 1992) and energy forward contracts (Rajaraman and Alvarado, 1998).

Electricity cannot appreciably be stored and system stability requires constant balance between supply and demand. Electricity generated at different times and locations are therefore not perfect substitutes, which is different for other commodities. The most similar commodity, natural gas, can be stored in special storage facilities and in pipelines. Natural gas in pipelines allows supply to differ from demand because the gas pressure in the pipelines may vary. Additionally, electricity flows according to Kirchoff's laws and transmission capacity constraints, which means that electricity may be transmitted from a high price location to a low price location.

The increased price volatility after deregulation has created a demand for derivatives. Derivatives allow market players to transfer risks to others who could profit from taking the risk, and have become a popular way of achieving price certainty against volatile electricity prices. Likewise, it is important for industry regulators and policymakers to understand the purpose of the instruments used to hedge against congestion risks.

Transmission congestion derivatives define property rights and are a mechanism to hedge congestion price risk. Property rights provide market players with the financial benefits associated with transmission capacity and facilitate efficient use of scarce resources. Property rights are also a mechanism to reward transmission investments. The rights will give investors a tradable contract in return. The ability to hedge congestion price is an important feature in facilitating an efficient electricity market.

When market players trade between different locations, they face the risk of paying a congestion fee for transferring electricity. The congestion fee from bus[2] (or node) i to bus j is defined as:

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[2] bus refers to a node in the transmission network

$$\text{Congestion fee} = q_{ij} \ (P_j - P_i) \qquad\qquad (1)$$

in which q_{ij} is the amount of transferred electricity from bus i to bus j and P is the local bus price. The congestion fee arises from the scarcity of transmission.[3] Congestion fees are zero when there is adequate transmission, but some lines in some systems may suffer from congestion much of the time. When transmission capacity is scarce, however, prices can become high. To some extent these prices are predictable, but they contain a significant random component that can be problematic for traders.

If a generator trades with load at the local bus, it is not charged for transmission, and can use a forward product to hedge the price uncertainty. If a generator trades with a distant load, and there is a chance of congestion, the trade is exposed to congestion price risk. This discourages trade because trading across a congested path in either direction will be risky (Stoft, 2002). This may enhance market power by decreasing the number of distant trades. To reduce such problems players can utilize financial instruments to hedge against transmission congestion.

In the following we illustrate these issues. Our aim is to give an overview about different contractual arrangements. We also develop optimal hedge ratios for the transmission congestion derivatives, which are not found in the literature.

FINANCIAL TRANSMISSION RIGHTS

The basic types of transmission rights are:
- Financial transmission rights (FTR[4]) obligation: right to collect payment from (or an obligation to pay) the price difference associated with transmission congestion between destination and origin for a specified contract quantity.[5]
- Financial transmission rights (FTR) option: right to collect payment from the price difference associated with transmission congestion

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[3] This ignores the charge for losses, which is almost never above 10% and is far more predictable.

[4] FTRs are also often called transmission congestion contracts (TCCs). For more background on FTRs see Hogan (2003) and Stoft (2002).

[5] The set of point-to-point obligations can be decomposed into a set of balanced and unbalanced (injection or withdrawal of energy) obligations. The unbalanced FTRs can be used to hedge against losses (Hogan, 2002b).

between destination and origin for a specified contract quantity. If the price difference is negative the payoff is zero.

- Flowgate rights (FGR[6]): constraint-by-constraint hedge that gives the right to collect payments based on the shadow price associated with a particular transmission constraint.
- Physical transmission rights (PTR): right or priority to physical transmission for a specified amount between two defined locations.

While forward contracts are used to hedge the temporal risk, transmission rights are used for hedging spatial risk.[7] Transmission rights are used mainly to facilitate trade in advance of the physical scheduling (usually done by a system operator a day in advance). Physical and financial transmission rights have different impacts on market power and on the electricity transmission system. Every transmission line at any time has a net directed power flow, which may consist of flows in both directions (both are fictitious). For FTRs only the net power flow matters, while for physical rights the directed power flow determines their feasibility.

Financial rights are only instruments for hedging against financial risk. Often they are provided by the ISO and are restricted in number by the network capacity calculations of the ISO that ensures that the ISO has sufficient revenues to cover the payments to FTR holders (Hogan, 1992). Provision of options is more restrictive because they do not create counterflows. The feasibility test can be complex and may require a central coordinator to produce a feasible set of FTRs.

Physical rights give the right to inject a certain amount of electricity at point i and withdraw it at point j. The holders are guaranteed scheduling for their rights. These rights can make withholding of transmission capacity possible and necessitate capacity release rules (use-it-or-lose-it principle), and are more restrictive than FTRs. Another type of physical right confers only a scheduling priority and is a less centralized and more flexible approach.[8]

An FTR obligation will entitle its owner to be paid the price difference between two buses times the contract quantity over a specified time period. This payment will net out any price risk associated with using that path (i.e., paying congestion fees) if the hedge is perfect. Such

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[6] In the earliest proposals, these rights were categorized as physical rights, but in the recent proposals the value of the FGRs are decided in the ISO settlements, and they do not require the parties to obtain all the FGRs needed prior to settlement.

[7] FTRs are usually also forward contracts, since they are hedges against future congestion prices.

[8] An example is firm transmission rights in California.

payments will be made regardless of the owner's actual usage of the transmission system. The payments under this right are therefore independent of the owner's physical use of the grid. Even if the congestion risk is hedged, traders will still be exposed to locational price signals and should still make efficient choices for generation and load. The mathematical formulation for the payoff is:

$$FTR = Q_{ij}(P_j - P_i) \qquad (2)$$

in which P_j is the bus price at location j, P_i is the bus price at location i and Q_{ij} is the directed quantity specified for the path from i to j. A perfect hedge is created by purchasing a contract quantity, Q, that equals the amount of electricity that is transferred between the two locations, q. An FTR may be acquired by either purchasing it in auctions or in the secondary markets, or by investing in transmission lines. In the auction, the benefit function of the buyer or seller is maximized. The benefit function is assumed to be concave and differentiable and is optimized subject to all relevant system constraints. The auction determines the allocated amount of FTRs to market players and market clearing prices. It is also a mechanism for reconfiguration of FTRs. Ideally, the auction price of an FTR obligation should equal the expected future congestion price. ISOs in the US are non-profit organizations and therefore must redistribute the FTR auction revenue if it is in excess of FTR payments. In PJM this is achieved through auction revenue rights (ARRs) to system users. The ARRs entitle the buyer of an FTR to a payment equal to the price of the FTR. Therefore, an ARR holder faces no risk in the FTR auction and its bidding strategy may change significantly. The holder will have an incentive to submit high bids for FTRs, to the disadvantage of   players without FTRs.[9]

Siddiqui et al. (2003) study the prices of FTRs in the New York market and find that the prices do not reflect the congestion rents for large exposure hedges and over large distances, and that the FTR holders pay excessive risk premiums. The authors argue that this may be due to the way the FTRs are defined with fixed capacity over a fixed period and high transaction costs for disaggregating them in the secondary market. Market players therefore consistently predict transmission congestion incorrectly for all other hedges other than the small and straightforward hedges. Also the large number of possible FTRs decreases price discovery. Pricing of FTRs is based on anticipated and feasible congestion patterns which may not be realized in the actual dispatch. This may make FTRs mispriced.

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[9] I am grateful for this insight from an anonymous referee.

However, the pricing of FTRs may be symptomatic of an immature market. Also, arbitrage of electricity prices may be impossible because of illiquidity, risk aversion and regulatory risks (Siddiqui et al., 2003)

Locational prices are needed before an FTR can be defined. These should depend on transmission congestion and perhaps losses. Typically, FTR obligations are forward contracts that are settled in the day-ahead market. Their payoff (assuming a fixed contract quantity) is dependent only on the bus prices, not on the actual power flow, and it may be positive, zero or negative as illustrated in Figure -1 for a 1 MW FTR obligation. Prices will change during the specified contract period, so the value of the total payment to the FTR holder is calculated by averaging a series of fluctuating locational prices.

An FTR obligation will have a negative value if the contract covers a path for which the price at bus i (injection) is higher than the price at bus j (withdrawal), $P_i > P_j$. This can happen because the acquired FTR is defined opposite to the prevailing direction, or because electricity on this path is flowing from a high to a low price bus. The first is highly desirable in a transmission system because it relieves congestion, while the second can exist in a meshed network. In either case, if the FTR of the trader more than covers the trader's transmission needs during slack periods, the trader may suffer an unpredictable financial penalty for owning the unused part of its right (Bushnell and Stoft, 1996).
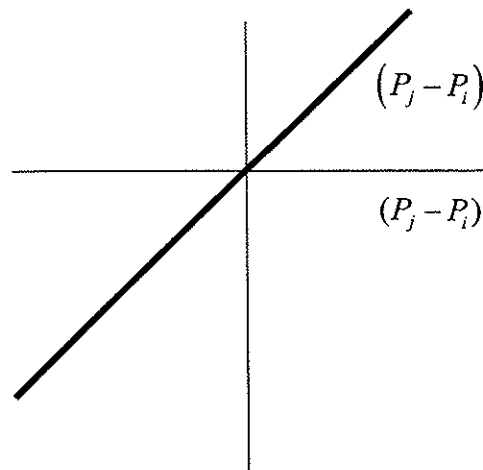

Figure -1. Payoff from a 1 MW FTR obligation.

A point-to-point transaction can also be hedged by purchasing a mix of FTRs. However, locational prices, congestion fees, and the values of FTRs are not defined until the dispatch occurs. Thus, the trader cannot be certain whether any mix of FTRs other than the point-to-point FTR provides a perfect hedge. FTRs may be more flexible if they are defined to and from central hubs because the buyer and seller then have one FTR for the same hub. When the buyer and seller enter into a contract they use two FTRs to hedge the congestion fee. The holders can then freely trade their contracts and make the secondary market more liquid. In general FTRs are more difficult to trade because of the large number of possible buses that can be used to define them. In an N-node network the possible number of FTRs is $1/2 \cdot N(N+1)$ for $N>2$. An FTR obligation is decomposable and has the following properties:

$$(P_j - P_j) = (P_{hub} - P_i) + (P_j - P_{hub}) \tag{3}$$

$$(P_j - P_i) = -(P_i - P_j)$$

FTRs can also be purchased as one-way options. In this case the holder is not responsible for negative payments that occur when the locational price difference is negative. The mathematical formulation for the payoff is:

$$\text{FTR\_ option} = max\ (Q_{ij}\ (P_j - P_i),\ 0) \tag{4}$$

Payments from an option are non-negative, and the option will have a clearing price greater than or equal to the price of an FTR obligation. The clearing price of an option is a function of the shadow price of each binding transmission constraint and it will never be less than zero for a buy bid. The payoff from a 1 MW FTR option is illustrated in Figure-2, a physical transmission right has a similar payoff.
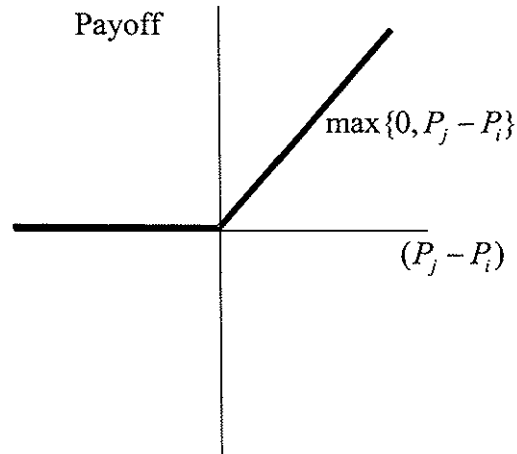
Figure -2. The payoff from a 1 MW FTR option.

If the objective is to fully and efficiently utilize the network, schedules that create counterflows are necessary, because they relieve congestion. Obligations also provide parties with transaction hedges against price uncertainty at generation and load buses. They work in favour of obligations. In the presence of counterflows, options issued by the ISO will not allow full hedging. The parties can then try to work out hedging arrangements in the private market. The FTR option does not have the same decomposition properties as the FTR obligation as demonstrated by:

$$\max(0, P_j - P_i) \neq \max(0, P_{hub} - P_i) + \max(0, P_j - P_{hub}) \qquad (5)$$

$$\max(0, P_j - P_i) \neq -\max(0, P_i - P_j)$$

Still another alternative is to use flowgate rights. The idea is that since electricity flows along many parallel paths, it may be natural to associate the payments with the actual flows. Key assumptions include a power system with few flowgates or constraints, known capacity limits at the flowgates and known power transfer distribution factors (PTDFs) that decompose a transaction into the flows over the flowgates. In practice, however, this may not be the case. The physical rights approach has been abandoned and a financial approach has been proposed in the literature (Hogan, 2002). The payoff from the FGRs is determined by taking the associated flowgate shadow price times the flowgate amount and totaling them for all lines k that are affected by the transaction between buses m and n (Equation).

$$FGR = \sum_{k} \eta_k f_k^f$$

$$\eta_k = \text{shadow price}$$

$$f_k^f = \left(\nabla K_y(Y^*, u^*)\right)_{km} Q = \text{the flowgate amount}$$

$$\left(\nabla K_y(Y^*, u^*)\right)_{km} = \text{the PTDF at the optimal operating point } (Y^*, u^*) \qquad (6)$$

$$\text{for a transaction between buses } m \text{ and } n \text{ over line } k$$

$$Q_k = \text{contract quantity}$$

The flowgate amount can take negative, zero or positive values.

To illustrate how FGRs can be used for hedging, an example from Hogan (2002) is provided in Figure -3. Here the lines 1-3 and 3-4 are constrained. The matrix of PTDFs is shown in Table -1.
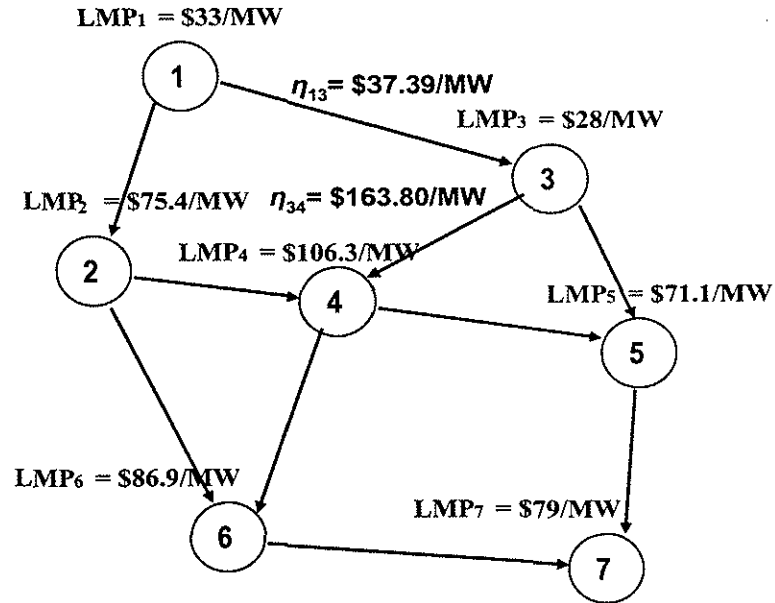
LMP₁ = $33/MW

Figure -3. Flowgate right example (Hogan, 2000).

Here a 100 MW transaction from 1 to 7 would pay:

$$\eta_{13} \cdot 100 \cdot 1/2 + \eta_{34} \cdot 100 \cdot 1/6 = \$37.39 \cdot 50 + \$163.80 \cdot 16.67 = \$4600$$

Thus the transaction can be hedged by buying 50 MW FGRs on 1-3 and 16.67 MW FGRs on 3-4. Similarly, a 100 MW transaction between 5 and 7 would pay:

$$\eta_{13} \cdot 100 \cdot -1/16 + \eta_{34} \cdot 100 \cdot 1/16 = -\$37.39 \cdot 6.25 + \$163.80 \cdot 6.25 = \$790$$

However, significant events occur if, for example, line 4-5 becomes congested or if the PTDFs change. A perfect hedge for the same transactions could be accomplished by purchasing a 100 MW FTR between 1 and 7 ((79-33)$/MW·100MW = $4600) or 5 and 7 ((79-71.1)$/MW·100MW = $790) that would pay exactly the same and would remain perfect if other lines became congested or the PTDFs changed.

In general, parties that want to be fully hedged should purchase a mix of FGRs that matches the distribution of flows from its transaction.[10] In a transmission network, the flows will be determined by the line impedances, and more than one flowgate (transmission constraint) may be affected. Flowgate proponents assert that trading is easy if there are few commercially significant flowgates, resulting in a limited set of FGRs and if the PTDFs change infrequently. This seems difficult to ensure in a dynamic power system where unanticipated transmission constraints may become binding (Hogan, 2000).

	Bus						
Line	1	2	3	4	5	6	7
1-2	1/2	-3/16	3/16	0	1/16	-1/16	0
1-3	1/2	3/16	-3/16	0	-1/16	1/16	0
2-4	1/6	17/48	-1/48	-1/6	-1/16	1/16	0
2-6	1/3	11/24	5/24	1/6	1/8	-1/8	0
3-4	1/6	-1/48	17/48	-1/6	1/16	-1/16	0
3-5	1/3	5/24	11/24	1/6	-1/8	1/8	0
4-5	1/6	11/48	5/48	1/3	-3/16	3/16	0
4-6	1/6	5/48	11/48	1/3	3/16	-3/16	0
5-7	1/2	7/16	9/16	1/2	11/16	5/16	0
6-7	1/2	9/16	7/16	1/2	5/16	11/16	0

Table -1. The matrix of PTDFs.

Although some ISOs sell transmission rights in their day-ahead markets, these markets are only approximations of the real-time congestion prices. A continuous market with a slowly changing price that traders can

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[10] This assumes that all constraints that could have been binding in the dispatch have been designated as flowgates, and that the ISO has made FGRs available for all flowgates. If some constraints have not been designated, but become binding, then there is no mechanism by which parties can purchase a perfect hedge. Some proposals for FGRs take this into account by not charging holders for the non-predicted constraints and instead socialize the costs.

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observe before trading may be needed. Afterwards, they can purchase transmission rights at a price close to the observed price. As yet, there are no such markets.

As a starting point for analysis, current congestion pricing allows market participants to make an educated guess about the financial consequences of future congestion. However, it should be emphasized that current congestion may wrongly estimate future congestion. For example, in the US the national load growth is projected to be around 1.8% per year, but at the moment there are few incentives for investments in the national grid. PJM awards builders of new transmission with ARRs that entitle the holders to collect some congestion rents. Federal Energy Regulatory Commission allows a return of rate on transmission assets of over 12% p.a., and is talking of increasing the allowable rate-of-return to over 16% in some areas. While it is impossible to predict future transmission congestion, it is possible to predict ranges for the closest months and years. One starting point is to use the market price of an FTR. Another is the utilization of a rigorous generation and transmission model for forecasting locational prices.

CONTRACTS FOR DIFFERENCES

The Nordic market (i.e., Nord Pool) has introduced Contracts for Differences (CfDs).[12] These financial instruments make it possible for the market players to hedge against the difference between the area (zonal) price and the System Price (the unconstrained price) in a future time period (Nord Pool, 2002). The area prices that are traded are: Oslo (NO1), Stockholm (SE), Helsinki (FI), Århus (DK1), and Copenhagen (DK2).

The forward and futures contracts traded at Nord Pool are with reference to the System Price. Producers are paid the area price for generation in their area. Consumers purchase load at their respective area price. Often, producers and consumers in different areas encounter situations of transmission congestion when the area prices differ from the System Price. They may also be exposed to significant financial risks

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[12] Here, the term Contract for Differences is different from the corresponding term used in the British market. In the Nordic region, CfDs are used to hedge against the difference between the two uncertain prices (area price and System Price), not as in the British market, where they hedge the difference between the spot price and a pre-defined reference price or price profile. The Nordic CfD is a locational swap, while the British CfD is settled based on the difference between the spot price and the reference price. When referring to CfD in the Nordic market this paper uses Nordic CfD.

associated with congestion fees for bilateral transactions in the Nordic countries that are calculated based on the difference between the area prices times the transferred quantity. Usually producers pay the fee, but parties can also make other arrangements.

The payment from the Nordic CfD is:

$$CfD = Q_i (AP_i - SP) \tag{7}$$

in which AP_i refers to the area price in area i, SP is the System Price, and Q_i is the contracted volume. Payments are calculated as the average of the difference between the daily area price and the System Price during the delivery period (a season or a year) times the contracted volume. From Equation (7) we see that each time the area price is higher than the System Price the holder receives a payment equal to the price differential times the contracted volume. Otherwise the holder must pay the difference.

The market price of a Nordic CfD can be positive, negative or zero (Kristiansen, 2004). CfDs trade at positive prices if the market expects that the area price will be higher than the System Price (a net import situation). CfDs trade at negative prices if the market expects an area price below the System Price (a net export situation).

A perfect hedge using forward or futures contracts is possible only when the area price and the System Price are equal. If forward or futures contracts are used for hedging, this implies a basis risk equal to the area price minus the System Price. To create a perfect hedge against the price differential:

1. Hedge the specified volume by using forward contracts.
2. Hedge against the price differential – for the same period and volume – by using CfDs.
3. Accomplish physical procurement by trading in the Elspot area of the holder of the contract.

Norway has adopted an area (zonal) price model to manage congestion in the day-ahead market. A charge equal to the difference between the System Price and low area price times the transferred volume (capacity charge) is imposed in the low price area, and a charge equal to the difference between the high area price and the System Price times the transferred volume is imposed in the high price area. Thus, withdrawals are charged in the high price area and compensated in the low price area.

The opposite is true for injections. However, it is impossible to hedge against price differences within Norway, because there is only one contract with reference to the area Norway 1 (Oslo). Shorter-term products and products for hedging directly against area price differentials are not available at the exchange. Nord Pool is considering listing CfDs with reference to Trondheim (NO2) and CfDs with shorter delivery periods such as weeks or months (Nord Pool, 2003). Nord Pool is also considering the listing of CfDs with reference to the German EEX price.

Kristiansen (2004) studied the prices of Contracts for Differences in the Nordic market and found that most of the contracts do not reflect the congestion rent. But there are also contracts that underestimate the congestion rent, resulting in a positive payoff to the holders. The Nordic CfDs are traded as forward contracts and do not have any connection to the congestion rent that the transmission system operator collects. The pricing of CfDs could be because the CfD market has only been in operation since November 2000 and therefore is immature. The majority of the results are in line with the pricing of futures at Nord Pool (Botterud et al., 2002).

TRANSMISSION RISK MANAGEMENT CONTRACTUAL ARRANGEMENTS

We analyze three different markets: a forward market (including a day-ahead market), a bilateral market, and the Nordic market. There are no deviations in the real-time market from the contracted volume. Hence, the market player does not participate in the real-time market and is paid the day-ahead price.

1. FORWARD MARKET

Assume that the generator sells electricity to a load at bus 2. The generator is paid the price at bus 2 and pays a congestion fee to the system operator so that the price it is effectively paid equals the price at bus 1. The load pays the price at bus 2. Other arrangements are also possible depending on the contract type. Assume that bus 1 is a surplus area and bus 2 is a deficit area. The price at bus 1 is therefore expected to be lower than at bus 2 as shown in Figure -4. Furthermore, assume that the contractual arrangements and transactions are for 1 MW.
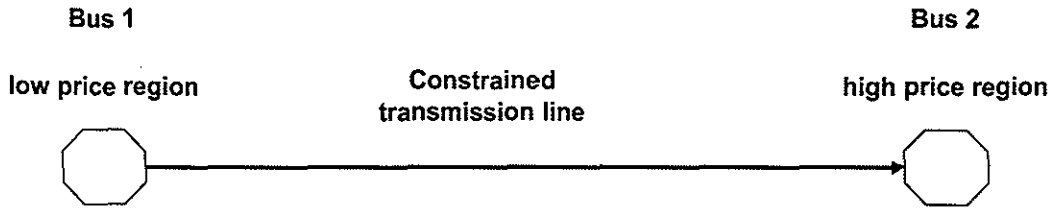
Bus 1   **Bus 2**

low price region   Constrained   high price region
                                    transmission line

Figure -4. Two buses connected by a constrained transmission line.

	Day-ahead market	Congestion fee	Total cash flow
Generator is paid:	P_2	$-(P_2-P_1)$	P_1
Load pays:	P_2		P_2

Table -2.
Consequences for the generator facing a congestion fee in the day-ahead market without an FTR.

The cash flow analysis shows the generator is indifferent between selling electricity at its local bus and at bus 2 (Table -2). To hedge the congestion fee, the generator buys an FTR obligation for the contracted volume. Its cash flow is shown in Table -3., where p_{FTR} is the contract price of the FTR.

	Day-ahead market	Congestion fee	FTR	Total cash flow
Generator is paid:	P_2	$-(P_2-P_1)$	$(P_2-P_1)-p_{FTR}$	P_2-p_{FTR}
Load pays:	P_2			P_2

Table -3
Consequences for the generator facing a congestion fee in the day-ahead market with an FTR.

The revenue of the generator will be dependent on the price at bus 2 and the price of the FTR. It avoids paying a congestion fee and is paid the price at bus 2 by purchasing an FTR. This arrangement is profitable if the contract price is less than the differences in the local day-ahead prices, $p_{FTR} < P_2 - P_1$.

	Day-ahead market	Congestion fee	Total cash flow
Generator is paid:	P_2	$(P_1 - P_2)$	P_1
Load pays:	P_2		P_2

Table -4.
Consequences for the generator arranging a sale in the low price area
(bus 2) in the day-ahead market without an FTR.

If the price at bus 1 is higher than at bus 2 and the generator has arranged a sale at bus 2, the generator receives compensation for relieving congestion equal to the congestion fee as shown in Table -4. Therefore it is indifferent to selling electricity at the local (high price) bus and the distant (low price) bus.

Table -5 illustrates the situation in which the generator receives compensation, but the FTR is an obligation and so the generator must pay the same amount to the seller of the right. Buying an FTR is profitable if the contract price is less than the difference in prices between locations 2 and 1.

	Day-ahead market	Congestion fee	FTR	Total cash flow
Generator is paid:	P_2	$(P_1 - P_2)$	$-(P_1 - P_2) - p_{FTR}$	$P_2 - p_{FTR}$
Load pays:	P_2			P_2

Table -5.
Consequences for the generator arranging a sale in the low price area
in the day-ahead market with an FTR.

Next, assume that a trader arranges to buy 1 MW at a price P_1 from the generator at bus 1 (low price) and sell it to the load at bus 2 for the price P_2 (high price). It also pays the congestion fee as shown in Table -6.

	Day-ahead market	Congestion fee	Total cash flow
Generator is paid:	P_1		P_1
Load pays:	P_2		P_2
The profit of the trader:	(P_2-P_1)	$-(P_2-P_1)$	0

Table -6.
Consequences for the trader in the day-ahead market without an FTR.

The trader does not profit when the line is congested. To hedge the congestion fee, it buys an FTR and receives a profit (or an expense) equal to that of the congestion fee minus the contract cost. If the price of the FTR is lower than the price differential between buses 2 and 1, this is a profitable trade as shown in Table -7.

	Day-ahead market	Congestion fee	FTR	Total cash flow
Generator is paid:	P_1			P_1
Load pays:	P_2			P_2
The profit of the trader:	(P_2-P_1)	$-(P_2-P_1)$	$(P_2-P_1)-p_{FTR}$	$(P_2-P_1)-p_{FTR}$

Table -7.
Consequences for the trader in the day-ahead market with an FTR.

1.1 HEDGING BY TAKING OPPOSITE POSITIONS IN THE FORWARD MARKETS

This hedging strategy requires that there are two energy forward markets with prices p1 and p2 in the two regions in which the trade is accomplished. The hedge gives the same payoff as the congestion fee, (P2 − P1). Assume that the contracted volume is 1 MW. The generator in region 1 can then enter into a contract agreement where it is long (buying) in the region of the load and short (selling) in its own region. The generator pays the congestion fee. This gives a combined cost equal to:

$$(p_2 - p_1) + (P_2 - P_1) - (P_2 - P_1) = (p_2 - p_1) \qquad (8)$$

in which p1 and p2 are the forward prices in the two regions. Parties have also agreed that the generator sells electricity to the load at price pC. The consequences are illustrated in Table -8..

	Forward market	Congestion fee	Day-ahead market	Total cash flow
Generator is paid:	$p_1 - p_2 + p_C$	$-(P_2 - P_1)$	$(P_2 - P_1)$	$p_1 - p_2 + p_C$

Table -8.
The cash flows of a generator from a bilateral trade while hedging against the congestion fee.

This contractual arrangement gives the generator a cash flow that is perfectly hedged. When there is congestion $(P_2 > P_1)$ the generator in region 1 will receive a net profit which may be higher than in its local forward market,[13] since it can sell electricity in region 2 at the fixed price p_C at a cost of $(p_2 - p_1)$.

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[13]This depends on the level of the forward prices in region 2 compared to the fixed contract price pC. If the contract price is higher than the forward price in region 2, the net profit will be higher. Conversely, when the contract price is lower than the forward price in region 2, the net profit will be lower.

1.2 HEDGING WITH OPTIONS

The advantage of an option is that it does not give a negative payoff. However, the price will be higher, since the market prices this into a premium. The payoff from a 1 MW option is:

$$\max(0, P_2 - P_1) = \begin{cases} (P_2 - P_1) & P_2 > P_1 \\ 0 & P_1 > P_2 \end{cases} \quad (9)$$

When the price at bus 2 is higher than at bus 1, the generator is assumed to pay the congestion fee.

	Day-ahead market	Congestion fee	Option	Total cash flow
Generator is paid:	P_2	$-(P_2 - P_1)$	$(P_2 - P_1)$ $-p_{option}$	$P_2 - p_{option}$
Load pays:	P_2			P_2

Table -9
Consequences for the generator facing a congestion fee in the day-ahead market when buying an option.

The generator has hedged the congestion fee as shown in Table -9. Its expected profit will be lower than through purchasing FTRs because the price of the option will be correspondingly higher. Consider the case where the price at bus 1 is higher than at bus 2 and the sale is conducted at bus 2 (Table - 10). The generator receives its price at bus 1 because it receives a rebate equal to the congestion fee for relieving congestion, but at the same time it has paid for an option with zero payoff ($P_1 > P_2$).

	Day-ahead market	Congestion fee	Option	Total cash flow
Generator is paid:	P_2	$-(P_2 - P_1)$	$-p_{option}$	$P_1 - p_{option}$

Table -10.
Consequences for the generator facing a congestion fee in the day-ahead market when buying an option and the price at bus 1 is higher than at bus 2.

1.3 THE BILATERAL MARKET

Traders must find each other and negotiate contracts. Consider two types of contracts: a standard bilateral contract and a British contract for differences (CfD). The British CfD makes it possible to hedge against the difference between the spot[14] price and a pre-defined reference price or price profile and can be written in several ways.

Assume that the generator and load have signed a bilateral contract of volume 1 MW without the benefit of a middleman. The price of the contract is PC. The generator pays the congestion fee and is paid the contract price. P1 is the day-ahead price at the bus of the generator. P2 is the day-ahead price at the bus of the load. First consider the case with the bilateral contract and no insurance as shown in Table -11.

	Bilateral market	Congestion fee	Total cash flow
Generator is paid:	P_C	$-(P_2-P_1)$	$P_C-(P_2-P_1)$
Load pays:	P_C		P_C

Table -11 Consequences for the generator paying a congestion fee in the bilateral market without an FTR.

By buying an FTR the generator will be compensated for the congestion fee as shown in Table -12. The FTR makes it possible to fix the price of transmission. The arrangement will be profitable if $p_{FTR} < P_2 - P_1$ which is the same condition as in the preceding cases.

	Bilateral market	Congestion fee	FTR	Total cash flow
Generator is paid:	P_C	$-(P_2-P_1)$	(P_2-P_1) $-p_{FTR}$	P_C-p_{FTR}
Load pays:	P_C			P_C

Table -12. Consequences for the generator with an FTR.

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[14] The spot price is assumed to be equal to the day-ahead price, since there are no deviations in contracted and delivered volumes. Originally the CfD was with reference to the spot price.

The second example considers a CfD where the generator pays for transmission. The situation is illustrated in Table -13.

Effect of CfD	Payment from load to generator
Generator pays for transmission:	$(P_C - P_2)$

Table -13. Generator pays for transmission CfD.

The generator is not hedged against locational price differences as illustrated in Table -14. The effect of using the CfD is that the load pays a fixed price for the electricity, while the generator receives a fixed price for electricity and pays the congestion fee. To hedge the congestion fee, the generator can buy an FTR as shown in Table -15.

	CfD	Spot market	Total cash flow
Generator is paid:	$(P_C - P_2)$	P_1	$P_C - (P_2 - P_1)$
Load pays:	$(P_C - P_2)$	P_2	P_C

Table -14
Cash flows to the parties resulting from using a CfD when the generator pays for transmission.

	CfD	Spot market	FTR	Total cash flow
Generator is paid:	$(P_C - P_2)$	P_1	$(P_2 - P_1)-p_{FTR}$	$P_C - p_{FTR}$
Load pays:	$(P_C - P_2)$	P_2		P_C

Table -15.
Cash flows to the parties resulting from using a CfD when the generator pays for transmission and has purchased an FTR.

In the next example, the trader pays the congestion fee, because it has agreed to buy 1 MW at bus 1 at a price f_1 and sell the power at bus 2 at a price f_2. However, since both the generator and load participate in the spot market, the trader must specify that the generator will pay it P_1 (the amount the generator is paid in the local spot market). The trader pays load P_2 (the amount the load pays in the local spot market). This trade constitutes two CfDs: trader pays generator $(f_1 - P_1)$ and load pays the trader $(f_2 - P_2)$. This arrangement is favorable when the generator and load want price certainty, and the trader wants to exploit profits from electricity trading. The trade is illustrated in Table -16.

	CfD	Spot market	Congestion fee/FTR	Total cash flow
Generator is paid:	$(f_1 - P_1)$	P_1		f_1
Load pays:	$(f_2 - P_2)$	P_2		f_2
Profit of the trader:	$(f_2 - f_1)$		$-(P_2 - P_1)$	$(f_2 - f_1) - (P_2 - P_1)$
Profit of the trader with an FTR:	$(f_2 - f_1)$		$-(P_2 - P_1)$ $+ (P_2 - P_1)$ $-p_{FTR}$	$(f_2 - f_1) - p_{FTR}$

Table -16
Cash flows to a trader providing two CfDs and at the same time paying the congestion fee.

As shown the trader is perfectly hedged against locational price differences by purchasing an FTR. This is profitable for the trader as long as the contract price is less than the difference in bus prices between the two locations.

1.4 THE NORDIC MARKET
Assume that there is a System Price (i.e., unconstrained price), and area (zonal) prices. Most financial contracts are referred to the System Price, while the generators are paid the local price for their production and the consumers pay their local area price. This means that the parties are left with a risk that the System Price and the local area price differ due to transmission congestion. According to the area price model, withdrawals

are charged in the high price area and compensated in the low price area. Injections are compensated in the high price area (B) and charged in the low price area (A). Congestion fees for bilateral transactions in the Nordic countries are calculated based on the difference between the area prices times the transferred quantity.

Assume a load has purchased a forward contract of volume 1 MWh[15] from the exchange at the price p_f and a CfD of the same volume at the price p_{CfD}. In addition it also accomplishes physical procurement by trading the same volume in its local spot area. The cash flow during the delivery period is shown in Table -17.

	Forward market	CfD	Day-ahead market	Total cash flow
Load pays:	p_f	p_{CfD}	$AP_B - SP$ $- (AP_B - SP)$	$(p_f + p_{CfD})$

Table -17
The cash flows of a load in the delivery period resulting from the purchase of a forward and a Nordic CfD.

It fixes the costs of purchasing electricity to the prices of the forward contracts and is therefore perfectly hedged against any uncertainties in spot prices.

Similarly, assume that a generator has sold a standard forward contract and a Nordic CfD, both with volume 1 MWh. Its cash flows are shown in Table -18. In this case the generator fixes its revenue to the prices of forward contracts.

	Forward market	CfD	Day-ahead market	Total cash flow
Generator is paid:	p_f	p_{CfD}	$AP_A - Q_s SP$ $- (AP_A - SP)$	$(p_f + p_{CfD})$

Table -18.
The cash flows of a generator in the delivery period resulting from the sale of a forward and a Nordic CfD.

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[15] At Nord Pool, the prices are quoted in NOK/MWh.

Another contractual arrangement is when a generator in area A enters into a contract to sell electricity to a consumer in another area B at the price P_C as shown in Table -19. The congestion fee is paid by the generator. In this market there are no FTRs available so the generator must use Nordic CfDs. A synthetic FTR is replicated by buying one CfD (long position) for the delivery area (B) and selling one CfD (short position) for the generation area (A). The payoff for 1 MWh is:

$$FTR = (AP_B - SP) - (AP_A - SP) = AP_B - AP_A \qquad (10)$$

As a result, the generator is able to hedge perfectly against the area price differential at a fixed cost of $(p_{CfDB} - p_{CfDA})$.

	CfDs	Bilateral contract	Day-ahead market	Total cash flow
Generator is paid:	$-p_{CfDB} + p_{CfDA}$	P_C	$(AP_A - AP_B)$ $+ (AP_B - AP_A)$	$P_C - p_{CfDB} + p_{CfDA}$

Table -19
The cash flows of a generator from a bilateral trade while hedging against the congestion fee.

1.5 OPTIMAL HEDGING

Traditionally, hedging can be done by entering an identical but opposite position to offset all risk. One replicates the risky asset by taking a short position in a forward instrument if the relationship between the prices of the two assets is linear. It can be shown (Hull, 2003) that the optimal hedge ratio for a player that wants to hedge its spot (or day-ahead) position (S) is to purchase the amount h^* of forward (F) contracts:

$$h^* = \rho_{SF} \frac{\sigma_S}{\sigma_F} \qquad (11)$$

in which σ is the volatility of the return on the assets and ρ is the correlation between the spot and forward returns. The returns of both the spot and the forward returns can be estimated from historical data. The calculation of the hedge ratio does not assume anything about the distribution of prices. However, in electricity markets, the spot prices are

often observed to have properties such as mean reversion, seasonality, spikes and a more complex probability distribution. The volatility of the spot price also fluctuates considerably over time. In electricity contract pricing it is usual to assume that the prices of futures and forward contracts are log-normally distributed, meaning that the natural logarithm of the return is approximately normally distributed. Spot prices may be approximated by a log-normal distribution. It is therefore possible to calculate the standard deviation of the natural logarithm of the return of the spot and forward prices[16] (see also Hull, 2003). Otherwise the standard deviation of the changes in prices can be used.[17]

The optimal hedge ratio can be calculated based on differences in locational prices or as in the Nordic system the difference between the area price and System Price. However, there should exist a liquid forward or futures markets where continuous rebalancing of the hedge could be performed. This may be difficult in many financial transmission rights markets.

The optimal hedge can be illustrated when the underlying asset is a price differential $P_{PD} = P_{AP} - P_{SP}$ between the area and System Price following the methodology utilized by Tanlapco et al. (2002). The purpose of this hedging is to insulate from price variations. Assume that the hedge is for one MWh and that the market player wants to trade at different locations. The value of the hedge (H) is:

$$H = P_{AP} - P_{SP} + h \ [F_{CfD,t-1} - F_{CfD,t}] \tag{12}$$

in which h represents the number of MWhs of CfDs that are used for hedging (i.e., the hedge ratio) while $F_{CfD,t}$ and $F_{CfD,t-1}$ are the prices of CfDs at time t and t-1 respectively. If h is negative, then the player buys forward contracts at time t. Conversely if h is positive it sells forward contracts at time t. A value h equal to 1 means that the company is fully hedged (i.e., riskless hedge). Hedging is performed in a two-period setting and the player plans to sell h of the closest (t-1) forward contract. At time t when the anticipated spot market transaction occurs, the player closes out its forward position by purchasing the same forward contract at time t.

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[16] Also called the log-returns. This method fails in case of negative or zero prices.

[17] Another possibility is to use models such as exponentially weighted moving average (EWMA), autoregressive conditional heteroscedasticity (ARCH), or generalized autoregressive conditional heteroscedasticity (GARCH). These models attempt to incorporate that volatility, and correlations vary over time. The GARCH model incorporates mean reversion, whereas the EWMA model does not.

This avoids physical delivery of the forward contract. The derivation of the optimal hedge is done in a minimum risk framework of a risk-averse company.[18] The mean and the variance of the hedge are shown as:

$$E[H] = E[P_{AP}] - E[P_{SP}] - hE[F_{CfD,t}] + hF_{CfD,t-1}$$
$$Var[H] = \sigma_{SP}^2 + \sigma_{AP}^2 + h^2\sigma_{CfD,t}^2 + 2h\rho_{SP,CfD,t}\sigma_{SP}\sigma_{CfD,t} \qquad (13)$$
$$-2h\rho_{AP,CfD,t}\sigma_{AP}\sigma_{CfD,t} - 2\rho_{SP,AP}\sigma_{SP}\sigma_{AP}$$

The price of the CfD is known at *t*-1 and therefore certain. σ is the standard deviation of the return of the price.[19] The variance is minimized with respect to the hedge ratio[20] when:

$$h^* = \frac{(\rho_{AP,CfD,t}\sigma_{AP} - \rho_{SP,CfD,t}\sigma_{SP})}{\sigma_{CfD,t}} = \frac{\rho_{APSP,CfD,t}\sigma_{APSP}}{\sigma_{CfD,t}} \qquad (14)$$

Here σ_{APSP} is the standard deviation of the return of the difference between the area and System Prices, and $\rho_{APSP,CfD,t}$ is the correlation between the area/ System Price differential and the CfD price. The greater the covariance between the spot and Nordic CfD prices, the higher the forward market position for every MWh to be sold in the spot market, all else being equal. Conversely, if the variance of the CfD prices is high, this tends to lower the CfD position. The hedge is riskless (*h*=1) when $\sigma_{CfD,t} = \rho_{APSP,CfD,t}\sigma_{APSP}$.

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[18] One reason why a risk-minimization framework is acceptable is that for a highly risk-averse agent, the problem of maximizing a mean-variance utility function collapses into a variance-minimization problem.

[19] If the log-returns are used, the standard deviation of these should be calculated.

[20] In the last equation we used:

$$\rho_{APSP,CfD,t}\sigma_{APSP} = \frac{COV(AP-SP,CfD_t)}{\sigma_{APSP} \cdot \sigma_{CfD_t}} \cdot \sigma_{APSP}$$
$$= \frac{COV(AP,CfD_t)}{\sigma_{AP} \cdot \sigma_{CfD_t}} \cdot \sigma_{AP} - \frac{COV(SP,CfD_t)}{\sigma_{SP} \cdot \sigma_{CfD_t}} \cdot \sigma_{SP} .$$

The corresponding hedge for an FTR would be similar:

$$h_{FTR}^{*} = \frac{(\rho_{P2,FTR,t}\sigma_{P2} - \rho_{P1,FTR,t}\sigma_{P1})}{\sigma_{FTR,t}} \tag{15}$$

in which $\rho_{P,FTR,t}$ is the correlation between the locational price and the FTR price, σ_P is the standard deviation of the return of the locational price, and σ_{FTR} is the standard deviation of the return of the FTR price. In practice, performing the optimal hedging strategy would require a liquid market where trades could be conducted whenever there was a need.

We can derive the corresponding hedge for two forward contracts for two different regions:

$$H = P_B - P_A + h_1[F_{B,t-1} - F_{B,t}] - h_2[F_{A,t-1} - F_{A,t}] \tag{16}$$

If h_1 is negative the trader buys forward contracts at time t and if h_1 is positive it sells contracts at time t. The opposite is the case for h_2. Similarly the mean and the variance of the hedge are:[21]

$$
\begin{aligned}
E[H] &= E[P_B] - E[P_A] - h_1 E[F_{B,t}] + h_1 F_{B,t-1} + h_2 E[F_{A,t}] - h_2 F_{A,t-1} \\
Var[H] &= \sigma_A^2 + \sigma_B^2 + h_1^2 \sigma_{FA}^2 + h_2^2 \sigma_{FB}^2 - 2\rho_{A,B}\sigma_A\sigma_B - \\
&\quad -2h_1\rho_{B,FB}\sigma_B\sigma_{FB} + 2h_2\rho_{B,FA}\sigma_B\sigma_{FA} + 2h_1\rho_{A,FB}\sigma_A\sigma_{FB} \\
&\quad -2h_2\rho_{A,FA}\sigma_A\sigma_{FA} - 2h_1 h_2\rho_{FA,FB}\sigma_{FA}\sigma_{FB}
\end{aligned} \tag{17}
$$

in which *FA* and *FB* are referred to time t.

The first-order conditions for optimality are:

$$\underset{h_1}{Min}\ Var[H] = 2h_1\sigma_{FA}^2 + 2\rho_{B,FB}\sigma_B\sigma_{FB} + 2\rho_{A,FB}\sigma_A\sigma_{FB} - 2h_2\rho_{FA,FB}\sigma_{FA}\sigma_{FB} = 0 \tag{18}$$

$$\underset{h_2}{Min}\ Var[H] = 2h_2\sigma_{FB}^2 + 2\rho_{B,FA}\sigma_B\sigma_{FA} - 2\rho_{A,FA}\sigma_A\sigma_{FA} - 2h_1\rho_{FA,FB}\sigma_{FA}\sigma_{FB} = 0$$

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[21] The hedges are derived with respect to time t.

The second derivatives with respect to h_1 and h_2 are positive, so a minimum is found. Solving for h_1 and h_2 gives:

$$h_1^* = \frac{1}{\rho_{FA,FB}\sigma_{FB}}\left\{\frac{1}{\sigma_{FA}^2(\rho_{FA,FB}^2-1)}[\sigma_{FA}^2(\rho_{B,FA}\sigma_B-\rho_{A,FA}\sigma_A)\right.$$
$$\left.+\rho_{FA,FB}\sigma_{FB}^2(\rho_{B,FB}\sigma_B+\rho_{A,FB}\sigma_A)]+\rho_{B,FA}\sigma_B-\rho_{A,FA}\sigma_A\right\} \quad (19)$$
$$h_2^* = \frac{1}{\sigma_{FA}\sigma_{FB}^2(\rho_{FA,FB}^2-1)}\left\{\sigma_{FA}^2(\rho_{B,FA}\sigma_B-\rho_{A,FA}\sigma_A)\right.$$
$$\left.+\sigma_{FB}^2\rho_{FA,FB}(\rho_{B,FB}\sigma_B+\rho_{A,FB}\sigma_A)\right\}$$

The optimal hedge ratios for forward contracts are more complex than for CfDs. There are more uncertainties to monitor and hedge against.

CONCLUSIONS

In this paper we have described different instruments for hedging against transmission congestion, and illustrated the use of financial transmission rights in the forward and bilateral markets. The Nordic market has been used to demonstrate the use of Contracts for Differences to hedge against transmission congestion. The cost of congestion (i.e., the congestion fee) between two locations is offset precisely by a higher price at one location. Similarly, selling to a low price location is offset by compensation. All trades between different regions then, are as profitable on average as local trades. To hedge against the congestion fee traders may purchase financial transmission rights or energy forward contracts if there are forward markets at both locations. If the contract price is less than the price difference between the high and low price location, the trade may be profitable.

If the objective is to minimize risks a perfect hedge may be non-optimal. We derive expressions for optimal hedges for Nordic CfDs, financial transmission rights, and energy forward contracts with respect to different locations.

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